Social Networks 12 (1990) 83–97
North-Holland

DETECTING ROLE EQUIVALENCE *

Ronald S. BURT
Columbia University

Hummell and Sodeur (1987) propose a practical solution to detecting role equivalence in social network data. The solution is very fast, equally applicable to symmetric and asymmetric relations, involves no iterative computing, and is now readily available as one of the equivalence options in STRUCTURE. Unfortunately, their paper is only available in German in a book published for their colleagues in Germany. The purpose of this brief note is to give their extremely useful idea wider exposure.

1. Introduction

In the mid-1970s, the release of practical methods for empirically detecting structural equivalence in social networks made an important contribution to the fast-expanding popularity of network analysis in empirical research. The categorical approach in White et al. (1976) and the continuous distance approach in Burt (1976) are often cited for proposing empirical research methods still popular for detecting the conditions of structural equivalence discussed in concept by Lorrain and White (1971). Social scientists were empowered with rigorous tools for studying empirical data on relationships to make inferences about status/role-sets implied by the data. The result was a much strengthened

* This work was produced as part of the Research Program in Structural Analysis housed at Columbia University's Center for the Social Sciences. I am grateful to Wolfgang Sodeur for calling my attention to the use of triad patterns for equivalence analysis and to Jurgen Hoffmeyer-Zlotnik for arranging my attendance at the ZUMA survey network data conference where I met with Professor Sodeur. Ronan Van Rossem wrote the core of the STRUCTURE subroutine that aggregates triad patterns. A copy of the second edition of STRUCTURE 4.1 for IBM microcomputers can be obtained at no charge by sending a self-addressed envelope (minimum size 7.5 by 10.5 inches) to The Research Program in Structural Analysis, Center for the Social Sciences, Columbia University, New York, NY 10027, U.S.A. You will receive a Command Booklet explaining how to run the program and a disk containing the program and a variety of example analyses. Please indicate whether your prefer a 5.25 or 3.5-inch DOS disk.

epistemic link between empirical research and the long tradition of social structural theory built around the status/role-set duality.

Two broad classes of applications have developed: (1) Structural equivalence is used most widely for structurally informative data reduction. Redundant patterns of relations are identified within and across networks. Two individuals are structurally equivalent to the extent that they have identical relations with others. When two individuals are structurally equivalent, therefore, they are involved in the same relation pattern and so need not be distinguished in a structural analysis. This is the justification for reducing a complex network of relations between individuals down to a density table of average relations between positions—and then to an image matrix in a blockmodel. The basic terms of this class of applications have been well discussed in several places. I remain partial to the reviews in Burt (1982: Chapter 2) and Burt and Minor (1983: Chapter 13). (2) Structural equivalence is used as a sometimes powerful replacement for cohesion (cliques) in social influence models. In this class of applications, structurally equivalent individuals in a network are predicted to act and think similarly because of using one another as a reference group (e.g. see Burt, 1982: Chapter 5; 1987, for review and argument).

The operational models drew criticism almost as quickly as they began to be adopted in empirical research. The models themselves, and their two broadly defined applications, were not criticized so much as critics were troubled by the presumption that the models captured the substantive meaning of status and role. Statuses are defined by relations between roles, not relations between individuals. By grounding structural equivalence in relations between individuals and analyzing those relations to detect roles, the operational methods of Boorman, Breiger, Burt and White lost much of the substantive meaning of status and role. This was the theme of what are in retrospect two key papers released in the late 1970s: Lee Sailer's (1978) article on structural relatedness in *Social Networks* and a paper presented by Michael Mandel and Christopher Winship in the Social Networks session of the 1979 annual meetings of the American Sociological Association. Sailer's paper reflected ideas under debate at the University of California at Irvine, ideas eventually formalized in a later *Social Networks* article on regular equivalence by Douglas White and Karl Reitz (1983). Mandel and Winship's paper reflected debate in the mid-1970s (sketched in a 1974 manuscript by Winship) among students working with Harrison

White at Harvard on generalizing structural equivalence. The Mandel and Winship paper circulated in manuscript to emerge in a much refined final form as a chapter on role equivalence in *Sociological Methodology* (Winship and Mandel, 1983). Winship's 1974 commentary later appeared in *Social Networks* (Winship, 1988).

These two extensions of structural equivalence have similarly important goals and have suffered from similar problems. Both are intended to enrich the substantive meaning of structural equivalence by defining equivalence in terms of relations between roles rather than individuals. I'll illustrate this issue in a moment with concrete examples. There is little disagreement over the point that both models capture the substantive meaning of status and role better than the popular operational definitions of structural equivalence.

At the same time, both models have suffered from the lack of a simple operational model. Both exist as mathematical models that are abstract well beyond the computational interests of most social scientists likely to use the models in empirical research (although Winship and Mandel's discussion is richly annotated with substantive motivation for their proposed concept). More importantly, the models have not been easily available for empirical research. Regular equivalence became available recently as an option in the UCINET microcomputer program and methodological applications have appeared (e.g. Faust, 1988; Doreian, 1987), hopefully presaging analyses in which the model is put to substantive use. Unfortunately, the iterative algorithm is relatively slow and troubled by symmetric relations in a network (Doreian, 1987).

A simple operational model is now available. In the mid-1980s, Hans Hummell and Wolfgang Sodeur (1987) were trying to link their work on the triad census with the ongoing structural equivalence debate. They hit upon a strikingly practical solution to detecting role equivalence. Their solution is very similar, but not identical, to Winship and Mandel role equivalence in direct and two-step indirect ties. It is intuitively simple and in computation similar to structural equivalence, making it useful in teaching. It is fast to compute, making it practical on microcomputers. It involves no iteration, and so avoids the possibility of convergence failures. It handles equally well symmetric and asymmetric relations. Finally, its computation as a Euclidean distance means that statements about role equivalence can be tested for their adequacy with the same principal component measures used to assess statements about structural equivalence. The model is available as an option in the STRUCTURE network analysis program, so the extension of structural equivalence to role equivalence is now readily available for substantive application with the diverse behavioral models contained in STRUCTURE—models predicting contagion, autonomy, power and network equilibria. My purpose here is to briefly explain Hummell and Sodeur's solution and present results contrasting it with standard structural equivalence results on the same networks.

2. Role equivalence from triad patterns

There are three steps to the Hummell and Sodeur solution: (1) express each individual's role in a network as a triad census, a pattern of relative frequencies with which kinds of triads describe the individual's orientation to others; (2) compute Euclidean distances between triad patterns to determine equivalence between roles; (3) proceed as usual in structural equivalence analyses—with cluster analysis and multidimensional scaling of the distances—to detect sets of individuals who are role equivalent. Role equivalence defines equivalence in terms of identical relations with roles while structural equivalence defines equivalence in terms of identical relations with specific individuals. By casting an individual's role as pattern of triads in which the individual is involved, and comparing individuals in terms of their respective triad patterns, Hummell and Sodeur generalize equivalence beyond having identical relations with specific individuals.

Figure 1 contains a table of 36 triad structures that distinguish ways in which an individual is oriented toward others in a network. The individual is ego and the triad contains two alters.

The ten rows of the table in Figure 1 distinguish patterns of relations between ego and the two alters. [1] For example, the first row contains

[1] Hummell and Sodeur (1987: 187) distinguish the 36 triads in Figure 1 with respect to their earlier work with the triad census. Their identification numbers for the triads reflect the logic of distinctions among all 64 triads possible in a triad census. I have organized the 36 triad types in Figure 1 to simplify their presentation—in terms of ego's relations and the inter-alter relation—for the many persons not familiar with the triad census model and so facilitate their diffusion. For readers interested in linking Figure 1 back to the tables in Hummell and Sodeur's discussion, here are the triad identification numbers in Figure 1 followed by their identification numbers in Hummell and Sodeur's discussion: 1–1, 2–2, 3–3, 4–4, 5–16, 6–5, 7–18, 8–6, 9–7, 10–17, 11–27, 12–28, 13–29, 14–30, 15–34, 16–31, 17–36, 18–32, 19–33, 20–35, 21–8, 22–9, 23–11, 24–12, 25–23, 26–13, 27–26, 28–14, 29–15, 30–24, 31–10, 32–19, 33–21, 34–20, 35–22 and 36–25.

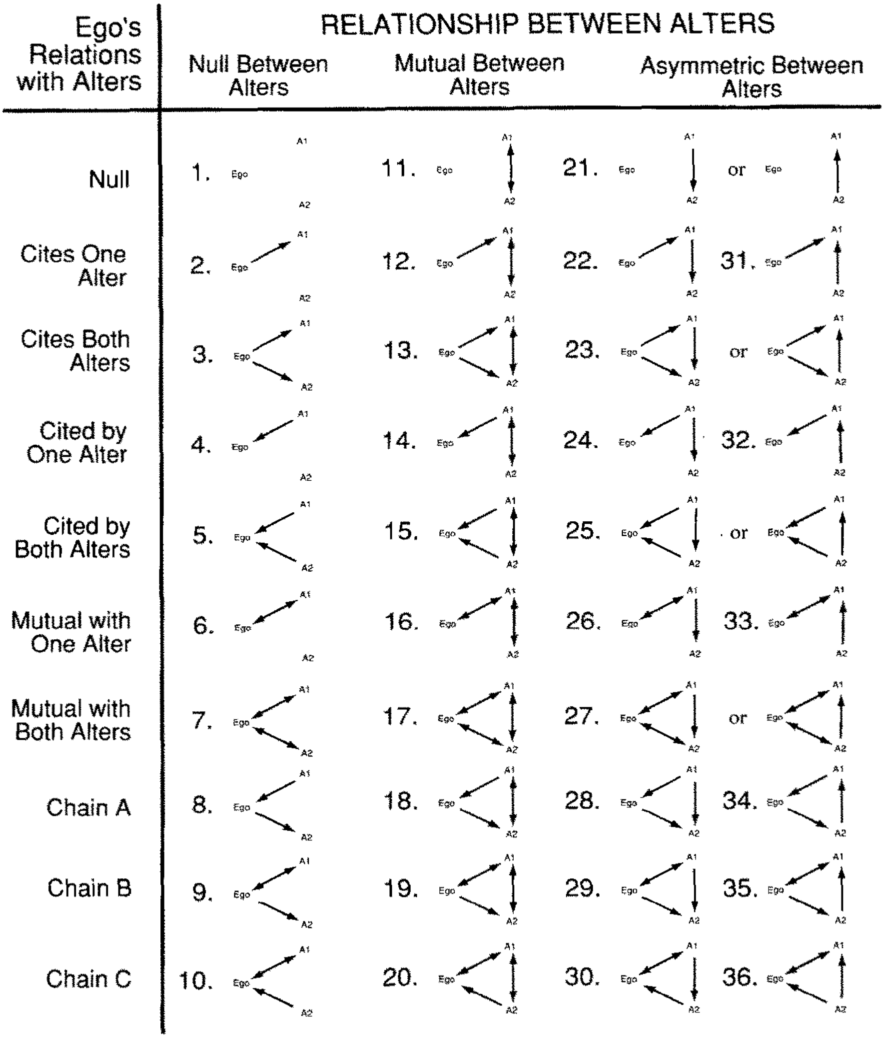

Fig. 1. Kinds of triads defining ego's role in a network.

triads in which ego has no connection with either alter. The seventh row contains triads in which ego sends relations to both alters and both reciprocate.

The three columns distinguish triads by the relationship between the alters. In the first column, there is no connection between them. In the second column, each alter directs a relation to the other. In the third column, only one of the alters directs a relation to the other. Two triad types are sometimes distinguished in the third column. This distinction occurs where ego has different relations with the two alters, therefore making it significant to know which alter has which relation with ego. In the first row, for example, there is no distinction between which alter cites the other. Ego has no relation with either alter, so their relations with one another do not affect ego's relation pattern. Illustrating the other condition, there is a distinction in the second row between two kinds of triads in the third column. Ego directs a relation to the first alter and has no contact with the second. If there is an asymmetric relation from the second alter to the first, then ego is one of two individuals directing relations to the first alter. If there is an asymmetric relation from the first alter to the second, then ego is in a chain of asymmetric relations from ego to alter one to alter two.

The important point in Figure 1 is that the identities of the alters are defined by the pattern of their relations with each other and ego. They are not identified as individuals. So, the microstructural "orientations toward others in a network" represented by the triad types in Figure 1 are component in roles. Figure 1 is an inventory of those components, and the relative frequency with which any one person, group, or institution as ego plays each of the triad types within a network defines ego's role in the network. [2] Two individuals then play the same role in a network to the extent that they are equivalently involved in the role components tabulated in their respective triad patterns. Specifically, each individual in a network of N individuals will be involved in $(N-1)(N-2)/2$ triads, each triad being one of the 36 types distinguished in Figure 1. [3] Let t_{jq} be the frequency with which individual

[2] On this note, Hummell and Sodeur propose to call the triad types in Figure 1 "triadic role types." In his letter to me commenting on the first draft of this paper, Sodeur defines a triadic role type as "an abstraction of a concrete triad which 'surrounds' one of its points (ego) and where the other two points (alters) may be regarded as interchangeable insofar, as only the constellation of links between them and ego is concerned. Again, the number of possible states is reduced: Respecting its type, the 'surrounding triad of ego' can only be in one of the 36 possible states." In this paper, I am focusing on the use of triad patterns to detect role equivalence. For more detailed discussion of the triad patterns as a development from earlier triadic measures of networks structure, see the original chapter by Hummell and Sodeur (1987).

[3] This assignment requires binary relations. Quantitative measures of relations are converted to binary in the triad subroutine in STRUCTURE by distinguishing zero and nonzero relations. Zero relations are coded as missing and nonzero relations are coded as present. Indirect contacts can be included as relations by requesting path distances before role distances are computed.

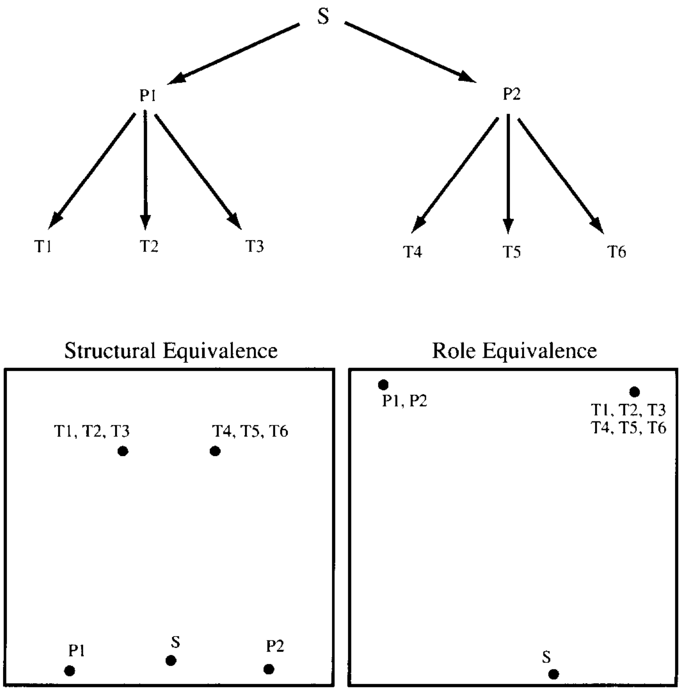

Fig. 2. Two-school supervision network (S = superintendent, P_i = principal, T_i = teacher).

j is involved in triads of type q. The triad pattern for individual j is then an array of 36 such frequencies; $t_{j1}, t_{j2}, t_{j3}, \ldots, t_{j36}$.

To illustrate, consider the sociogram in Figure 2. This is a variation on the educational hierarchy used as numerical illustration by Winship and Mandel (1983). One superintendent supervises two principals, each of whom supervises three teachers. Each individual in the nine-person network is involved in 28 triads (i.e. $7*8/2$).

Consider the role of principal. Being a principal involves seven of the 36 kinds of triads. The principal is involved in three triads of type 1 with the teachers in the other principal's school. She doesn't supervise them and they don't supervise one another. She is involved in 12 triads of type 2 created by her supervision ties to her teachers and their lack of supervision ties to the principal and teachers in the other school. She is involved in three triads of type 3 with the teachers in her own school. She supervises them and they don't supervise one another. She is involved in three triads of type 8 created by the indirect supervision she transmits from the school superintendent. The superintendent supervises her and she supervises her teachers, none of whom is supervised directly by the superintendent. She is involved in three triads of type 21 with the teachers of the other school through their principal. She neither supervises the other principal nor the teachers in the other school, who are supervised by their principal. Finally, she is involved in one triad of type 24. She and the other principal are supervised by the superintendent and neither principal supervises the other.

In sum, the role of being a principal in this network is described by the following pattern of triad frequencies (3 of type 1, 12 of type 2, and so on):

3 12 3 3 0 0 0 3 0 0 0 0 0 0 0 0 0 0 0 0 3 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0

In the same way, the role of principal in the second school is described by the following triad pattern:

3 12 3 3 0 0 0 3 0 0 0 0 0 0 0 0 0 0 0 0 3 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0

the role of superintendent is described by the following triad pattern:

15 6 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 6 0 0 0 0 0 0 0 0 0 0 0 0 0 0

and the roles of the six teachers are described by the following six triad patterns:

17 0 0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 0 0 2 0 0 0 0 0 0 0 1 0 0 0 0

17 0 0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 0 0 2 0 0 0 0 0 0 0 1 0 0 0 0

17 0 0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 0 0 2 0 0 0 0 0 0 0 1 0 0 0 0

17 0 0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 0 0 2 0 0 0 0 0 0 0 1 0 0 0 0

17 0 0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 0 0 2 0 0 0 0 0 0 0 1 0 0 0 0

17 0 0 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4 0 0 2 0 0 0 0 0 0 0 1 0 0 0 0

Notice that there is no indication of which individuals are connected by relations, just the relations with and between alters as a role in a triad. Specific individuals have been abstracted out of the relation pattern defining each individual's position in the network.

Role equivalence is immediately obvious. The two principals are involved in identical triad patterns. Their relation with specific individuals are very different—they are not structurally equivalent. However, they have identical relations with the other roles in the network and that fact is apparent from the identical triad patterns in which they are involved. Similarly, the six teachers are involved in identical triad patterns.

As a straightforward measure of differences between triad patterns, Hummell and Sodeur use Euclidean distance. This is not only useful for its simplicity, but for the analogy it creates with structural equivalence where Euclidean distance normalized in various ways (usually controlling for means and/or variance in relation patterns, Burt 1988) is the standard measure used to define equivalence. With respect to triad patterns, the distance between the role that individual j plays in a network and the role that individual i plays in the network is given as follows: [4]

$$d_{ij} = \left[\sum_q \left(t_{jq} - t_{iq} \right)^2 \right]^{1/2},$$

where q varies from 1 to 36 across the kinds of triads in Figure 1 as illustrated in the above triad patterns. When this distance is zero, individuals i and j are role equivalent under a strong criterion. They are decreasingly equivalent as d_{ij} increases.

The graphs in Figure 2 are multidimensional scalings of structural equivalence distances (defined in the usual way from differences in relations with specific individuals) and role equivalence distances as defined above. The scalings are a fairly close fit to the distance data, using the SYSTAT implementation of Kruskal's algorithm with a linear fit between observed and scaled distance. Equivalent individuals are right on top of one another, appearing as a single dot in the graphs, and distances indicate the degree of non-equivalence between individu-

[4] Two points should be noted here. First, Hummell and Sodeur (1987: 189) normalize distances by their upper limit (since the metric and upper limit of the triad frequencies are known). I have not, principally for simplicity. The distances are only meaningful relative to one another so any constant can be used here without affecting conclusions from the typically nonmetric procedures used to detect equivalence from distance data. Second, the triad frequencies can get quite large in large networks. However, only the relative magnitude of frequencies compared between triad patterns is essential to measuring role equivalence. To simplify program output, patterns are expressed as percentages in STRUCTURE before distances are computed. Each element in a triad pattern ranges from 0 to 100 and 141,42 is the maximum distance possible between two patterns. The role of principal in Figure 2, for example, is 11% null triads, 43% triads of type 2, and so on.

als. Under structural equivalence, there are five positions in the network: the superintendent, the principal of the first school, the teachers in the first school, the principal of the second school and the teachers in the second school. The teachers within each school are structurally equivalent because they have identical supervision from their principal, no supervision from the superintendent, and no contact with the other school. The principals are not equivalent to each other because they supervise different groups of teachers. In contrast to structural equivalence, which reflects vertical and horizontal dimensions of organization, role equivalence only reflects the horizontal. There are only three roles in the network: the superintendent, the principals, and the teachers. A quick inspection of the above triad patterns shows how individuals within each role are identical. The three roles are roughly equidistant from one another, with the superintendent—who is supervised by no one—furthest from the other roles.

3. Other examples

Figure 3 shows role equivalence in a hierarchy of symmetric relations. This is the illustration that Doreian (1987) used to discuss the difficulty that symmetric relations pose for regular equivalence. Regular equivalence reduces the whole network to a single role instead of distinguishing the four layers of roles. Notice in the graph to the right of Figure 3 that the four roles are appropriately distinguished. Notice also that the top roles are more similar to each other than either is to the bottom roles, and that the two roles between other layers (B–C, and D–E–F–G) appear to the left of the graph while the two roles at the ends of the hierarchy appear to the right. Structural equivalence provides a closer reading of the structure, but in the process obscures role similarities. The right-hand side of the structure is at the top of the graph at the bottom of Figure 3 and the left-hand side of the structure is at the bottom of the graph. The role equivalent broker roles B and C are at opposite ends of the structural equivalence graph. The role equivalent broker roles D, E, F and G are equidistant from one another in the four corners of the structural equivalence graph.

Figure 4 illustrates the point that structurally equivalent individuals are role equivalent. Role equivalence is a strategy for expanding equivalence classes to contain more individuals. There are two strong

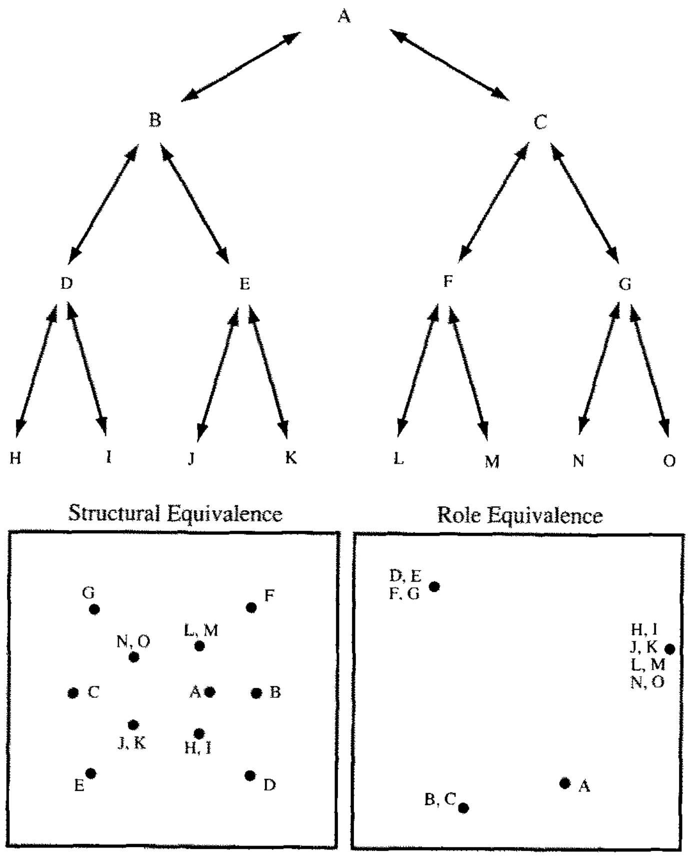

Fig. 3. Doreian's symmetric hierarchy.

components in the network: the triad J–K–L, and the four-person clique F–G–H–I. Structural equivalence equals, extends and restricts groupings by cohesion. The triad of mutually citing people is also structurally equivalent and appears at the bottom of the structural equivalence graph in Figure 4. They have identical relations beyond their group. Cohesion is extended with the addition of persons A–B–C–D as a position. They have no relations with each other and so are not a cohesive group. However, they have identical relations with everyone in the network and so are structurally equivalent. Cohesion is restricted with the disaggregation of the four-person clique. Although strong relations connect all people in the clique, persons H and I are

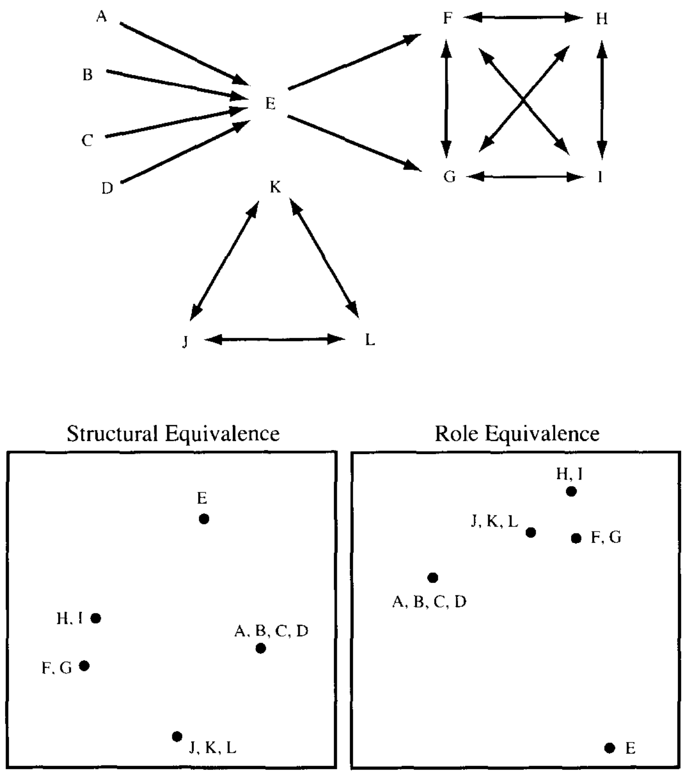

Fig. 4. Illustrative advice network.

only connected to other members of the clique, while persons F and G are cited by an outsider, making F and G non-equivalent to H and I. A quick comparison of the two graphs in Figure 4 shows that the five positions of structurally equivalent people are also positions of role equivalent people, although the relative distances between positions are different.

Figure 5 illustrates some of the subtleties captured by role equivalence. This is the network used by Hummell and Sodeur (1987) to illustrate their discussion. A and D are leaders in the network and role equivalent. The intermediaries between them, B and C, are role equivalent. Their four subordinates, E, G, H, and J, are role equivalent even though they don't answer to the same leaders. Finally, F and I are role equivalent across the two groups as followers who do not have direct

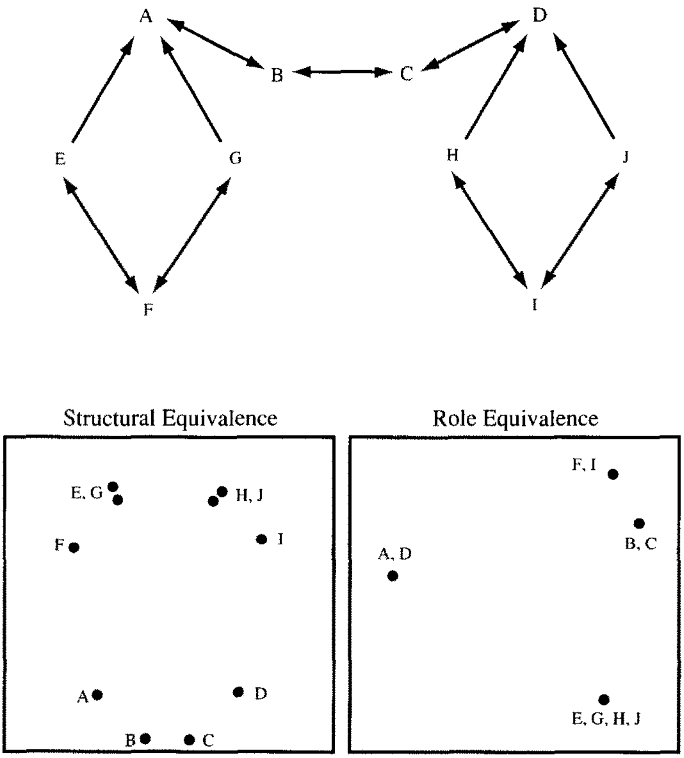

Fig. 5. Hummell and Sodeur illustrative network.

contact with the leaders A or D. Here again, structural equivalence splits the structure vertically and horizontally. The leaders and their followers are at the bottom of the structural equivalence graph and the followers are at the top. The people associated with leader A are at the left of the graph and those associated with leader D are at the right.

Finally, Figure 6 illustrates one important way in which the Hummell and Sodeur solution is not identical to Winship and Mandel's role equivalence model. In some ways this is a virtue, but it is clearly a deviation from the Winship and Mandel model; and one explicitly rejected, judging from Winship's early consideration of triad patterns as a basis for defining role equivalence (Winship, 1988: 217). The volume of relations in a role affects equivalence between roles when triad frequencies are the basis for equivalence. For example, the leader of a large group will not be equivalent with the leader of a small group.

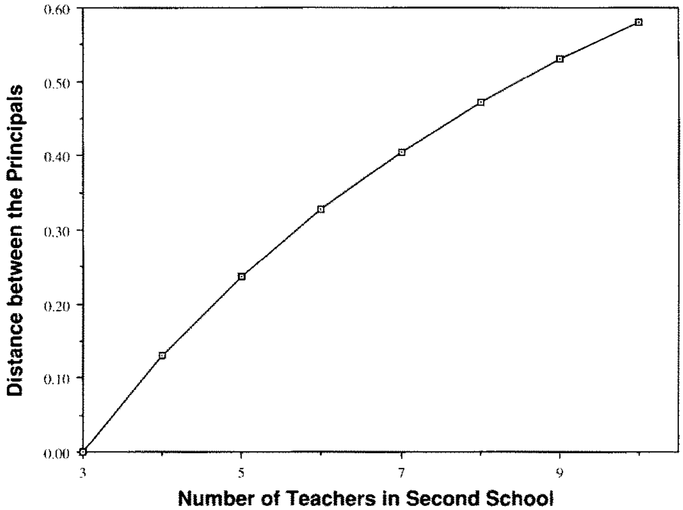

Fig. 6. Role distance between the principals in the two-school supervision network in Figure 2 as the second school expands (triad pattern role equivalence is affected by the volume of relations in a role).

In Figure 2, the two principals are role equivalent. They are separated by zero distance. If one school expands the number of teachers employed, however, the principals become increasingly distant. The graph in Figure 6 shows how the distance between the two principals increases as the second principal's school expands with the addition of more teachers. In the school system used as illustration in Winship and Mandel (1987: 319), there are two teachers in the first school and three teachers in the second. Winship and Mandel's model identifies the two principals as role equivalent under a strong criterion (p. 330). The Hummell and Sodeur solution does not—although the two principals are clearly more equivalent to each other than they are to anyone else. [5]

[5] Patrick Doreian proposed a simple way of eliminating relationship volume from the role distance measure. Replace nonzero frequencies t_{jq} with a 1. The Euclidean distance between two triad patterns of such binary data then measures the extent to which two individuals are involved in the same kinds of triads in a network-without measuring the extent to which they are equally involved in each kind of triad. This measure of role distance is zero between the two principals in Figure 6 for any number of teachers in the second school. This measure is available as an undocumented option in compilations of STRUCTURE dated March, 1989 or later. If you put a "1" in column 13 of a POSITIONS command requesting a role equivalence analysis, triad frequencies will be reduced to binary data and relationship volume is eliminated from the analysis.

In closing, Hummell and Sodeur's solution to detecting role equivalence is a powerful, useful and readily available addition to our tools for applied network analysis. At minimum, their contribution should encourage empirical and theoretical work making explicit its connection with White and Reitz's regular equivalence model and Winship and Mandel's role equivalence model. Hopefully, the ready availability of the model will encourage a broader audience of social scientists to use role equivalence in their substantive research.

References

Burt, R.S.
1976 "Positions in networks." *Social Forces 55*: 93–122.
Burt, R.S.
1982 *Toward a Structural Theory of Action*. New York: Academic Press.
Burt, R.S.
1987 "Social contagion and innovation". *American Journal of Sociology 92*: 1287–1335.
Burt, R.S.
1988 "Properties of structural equivalence measures derived from sociometric choice data." *Social Networks 10*: 10–28.
Burt, R.S. and M.J. Minor (eds.)
1983 *Applied Network Analysis*. Beverly Hills: Sage Publications.
Doreian, P.
1987 "Measuring regular equivalence in symmetric structures." *Social Networks 9*: 89–107.
Faust, K.
1988 "Comparison of methods for positional analysis: structural and general equivalence." *Social Networks 10*: 313–342.
Hummell, H. and W. Sodeur
1987 "Strukturbeschreibung von positionen in sozialen beziehungsnetzen." In Pappi, F.U. (ed.), *Methoden der Netzwerkanalayse*. Munich: Oldenbourg.
Lorrain, F.P. and H.C. White
1971 "Structural equivalence of individuals in social networks." *Journal of Mathematical Sociology* 1: 49–90.
Sailer, L.D.
1978 "Structural equivalence: Meaning and definition, computation and application." *Social Networks 1*: 73–90.
White, D.R. and K.P. Reitz
1983 "Graph and semigroup homomorphisms on networks of relations." *Social Networks 5*: 193–234.
White, H.C., S.A. Boorman and R.L. Breiger
1976 "Social structure from multiple networks. I. Blockmodels of roles and positions." *American Journal of Sociology 81*: 730–780.
Winship, C.
1988 "Thoughts about roles and relations: an old document revisited." *Social Networks 10*: 209–231.
Winship, C. and M. Mandel
1983 "Roles and positions: a critique and extension of the blockmodeling approach." In Leinhardt, S. (ed.), *Sociological Methodology 1983/1984*. San Francisco: Jossey-Bass.