*Positions in Networks**

RONALD S. BURT, *University of California, Berkeley*

ABSTRACT

The existence of an actor as a set of asymmetric relations to and from every actor in a network of relations is specified as the position of the actor in the network. Conditions of strong versus weak structural equivalence of actor positions in a network are defined. Network structure is characterized in terms of structurally nonequivalent, jointly occupied, network positions located in the observed network. The social distances of actors from network positions are specified as unobserved variables in structural equation models in order to extend the analysis of networks into the etiology and consequences of network structure.

We are each nested in a cacophony of relations with other actors in society. These relations serve to define our existence in society. We are who we are as a function of our relations to and from other actors in society. With the growth of technology and its concomitant division of labor, the determination of actors in society as a function of their relations with other actors is likely to increase rather than decrease. The problem for the social scientist then becomes one of conceptualizing the patterns of relations between an actor and the social system in which he exists in a manner optimally suited to explanation.

Within the total set of all relations which link an actor to other actors in a social system, there are subsets of similar relations. There are economic relations linking the actor to specific other actors. There are relations of friendship, relations of kinship, and relations of status. There are political relations linking the actor to other actors. The list has no end. Each of these types of relations among actors in a social system serves to define a network of relations among the actors. This paper elaborates a conceptualization of networks of relations among actors in a system which simultaneously captures the basic characteristics of the structure in an observed network of relations and easily lends itself to the investigation of the etiology and consequences of that structure through the use of structural equation models.

The central idea in the conceptualization is that of a position in a network, the specified set of relations to and from each actor in a system. The extent to which two actors jointly occupy the same network position is treated as the social distance between them and is specified as a scalar value. Two or more actors jointly occupy the same network position when they have similar relations to and from each actor in the network. Actors jointly occupying the same network position are discussed as

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being structurally equivalent in Section 2 and definitions of strong versus weak equivalence are specified then illustrated. Based on the discussion in Sections 1 and 2, Section 3 proposes that the structure in an observed network exists as a pattern of relations among $M + 1$ sets of actors who together form the system of actors being considered. The M sets of actors are each composed of multiple actors who jointly occupy structurally equivalent positions in the network. Actors in different sets occupy structurally nonequivalent positions. The M jointly occupied positions exist as four elementary types of network positions outlined in Section 3. The remaining set is a residual category of actors among whom there is no network position occupied by more than two actors. Sections 2 and 3 describe, primarily, the structure in an observed network of relations. Section 4 moves ahead by specifying network positions in structural equation models as unobserved variables in order to investigate the etiology and consequences of occupying different types of positions in different types of networks.

SOCIAL DISTANCE AND ACTORS AS POSITIONS IN NETWORKS

Given a system composed of N actors and one or more networks of relations among the actors, there are two perspectives from which the intensity of relation between two actors within a single network can be viewed: (1) from the perspective of the two actors as a dyad which is only secondarily associated with the overall network, or (2) from the perspective of the two actors as elements of the overall network. The former can be discussed in terms of an asymmetric *individual distance* from one actor to another. The latter can be discussed in terms of a symmetric *social distance* between the two actors in terms of their respective positions in the network.

Let Id_{ij} be the individual distance from actor i to actor j where Id_{ij} can be operationalized as any of a variety of measures of the directed, asymmetric relation from i toward j qua individual entities such as: (1) a measure of the difference between a profile of characteristics of i (e.g., i's values, socioeconomic characteristics, or aspirations), and i's perception of actor j's profile on the same characteristics;[1] (2) a measure of the co-occurrence of actors i and j in archival records where j initiates action while i is the object of action;[2] (3) the presence or absence of a sociometric choice link from actor i to actor j;[3] (4) the minimum number of sociometric choice links required for actor i to reach actor j;[4] (5) the minimum weighted sociometric choice link distance required for actor i to reach actor j;[5] (6) the normalized minimum number of sociometric choice links required for actor i to reach actor j.[6] Although the six operationalizations of individual distance do not measure the same qualitative idea, they all reflect an intensity of relationship from one actor toward another as a dyad, a pair, of actors. The other $N - 2$ actors in the network are ignored except in the last three operationalizations which consider only as many other actors as are necessary to complete a chain of sociometric choice links from actor i to actor j.[7]

When two actors exist in a network with several additional actors, an integral aspect of the intensity of relationship between them consists of their different relationships with each of the other actors in the network (see research on reference group influence on individual perception, e.g., Tajfel; and research on the motivation of actors to avoid dissonant situations by maintaining balanced social relations, e.g., references in note 10). For example, if one actor is a member of a clique which excludes the other actor, then they will have a greater social distance between them than they would if both actors were members of the same clique—even if the individual distances between them were the same under both circumstances. Further, the distance between the two actors would be larger still if each actor were a member of a clique which excluded the other actor.

For each of the $N(N-1)$ asymmetric relations between pairs of actors in the system, there is a value of Id_{ij} (assume Id_{ii} equals 0). An (N by N) matrix of individual distances among the actors can therefore be assembled. Let ID_{nn} refer to this matrix. Consider the ith row and column vectors of this (N by N) matrix. The row vector contains N elements which describe actor i's relation toward every actor in the network. The column vector conatins N elements which describe the relations of every actor in the network toward actor i. *Together the* i*th row vector and* i*th column vector in* ID_{nn} *completely describe the relationships of actor* i *with the actors in the network. If the two vectors are combined into a single* (2N *by* 1) *vector* (N *elements from* ID_{in} *and* N *elements from* ID_{ni}), *the new vector,* ID_i*, *can be discussed as the structural location of actor* i *in the network.* ID_i* *is the "position" of actor* i *within the network being analyzed.* ID_i* *defines actor* i *in terms of his relations to all other actors in the system as they are present in the network being analyzed.*

Conceptualizing two actors solely as elements in a network, the social distance between them can be given as the distance between their respective network positions. Such a distance has a simple interpretation geometrically as the Euclidean distance between the actors i and j where actor i is defined by ID_i* and actor j is defined by ID_j*. *The social distance between actors* i *and* j *can therefore be given as the square root of the sum of squared differences between corresponding elements of* ID_i* *and* ID_j*:[8]

$$d_{ij} = d_{ji} = \sqrt{\left[\sum^{N}_{k=1}(\mathrm{Id}_{ik} - \mathrm{Id}_{jk})^2 + \sum^{N}_{k=1}(\mathrm{Id}_{ki} - \mathrm{Id}_{kj})^2 - \mathrm{Id}_{ij}^2 - \mathrm{Id}_{ji}^2\right]}, \qquad (1a)$$

$$= \sqrt{[(\mathrm{ID}_i* - \mathrm{ID}_j*)' (\mathrm{ID}_i* - \mathrm{ID}_j*) - \mathrm{Id}_{ij}^2 - \mathrm{Id}_{ji}^2]}. \qquad (1b)$$

In contrast to the operationalizations of individual distance, social distance is a symmetric measure (i.e., $d_{ij} = d_{ji}$).[9] Further, although alternative operationalizations of individual distance can vary across research applications, the specification of equation (1) is invariant since social distance only has nominal meaning.[10] It is defined in terms of logic without reference to empirical information and therefore generalizes to any empirical situation. Social distance derives empirical meaning from whichever operationalization of individual distance is selected. (See Hempel's a, 654–66; b, 101–2, 197–213 discussion of empirical versus nominal meaning and Braithwaite's, 50–2, 79–84 related discussion of direct versus indirect meaning).

STRUCTURALLY EQUIVALENT POSITIONS IN A NETWORK

Actor j's position in a network is defined by ID_j^*, the ($2N$ by 1) vector of individual distances from actor j to each of the N actors and from each of the N actors to actor j. Given that actors have different relations with one another as a function of their interests in each others' resources such as finances, prestige, and charm, it is to be expected that positions of actors in a network will be differentially similar to one another. Two actors occupy the same position in a network when they have the same relations to and from each actor in the network. Such a pair of actors can be discussed as occupying structurally equivalent positions in the network. *More generally, a set of actors can be discussed as occupying structurally equivalent network positions when their relations with all actors in the network are identical so that:*

$$\mathrm{ID}_i^* = \mathrm{ID}_j^* = \mathrm{ID}_k^*, \tag{2a}$$

which means that actors i, j *and* k *occupy structurally equivalent positions in a network when the social distances among them equal zero:*

$$d_{ij} = d_{ik} = d_{jk} = 0. \tag{2b}$$

Since it requires that every element of ID_i^* *and* ID_j^* *be identical if actors* i *and* j *are to be discussed as structurally equivalent, equation (2) is a definition of strong structural equivalence.*[11]

When dealing with actual networks of relations, the strong definition of equivalence has little utility since there are likely to be minor differences between structurally equivalent positions due to sampling variability, errors of observation, and/or theoretically trivial differences between actors. For applied research therefore, it is convenient to relax equation (2) to a weak definition of structural equivalence. *A weak definition of structural equivalence of actor positions asks that* d_{ij}, *the social distance between actors* i *and* j, *be less than some criterion distance based on the distances among structurally nonequivalent positions in order for actors* i *and* j *to be treated as occupying structurally equivalent positions. In other words, actors* i and j *occupy structurally equivalent positions in a network under a definition of weak equivalence when:*

$$\mathrm{ID}_j^* \simeq \mathrm{ID}_j^*, \tag{3a}$$

which can be stated as

$$d_{ij} \leqslant \alpha, \tag{3b}$$

where alpha is a criterion distance based on the distances among the structurally nonequivalent positions in a network. The social distances among actors in struc-

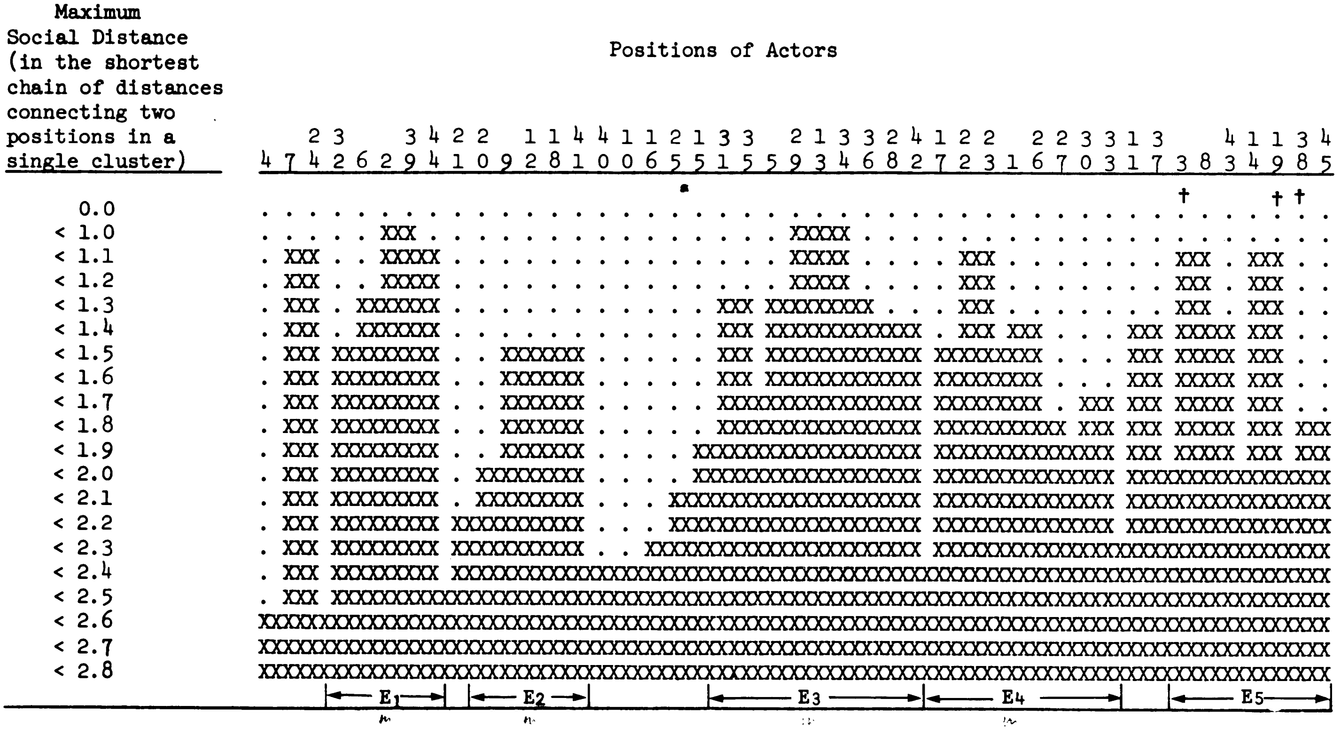
turally equivalent positions under the definition of weak equivalence is less than some arbitrary criterion, alpha.[12]

While the specification of a value of alpha for all networks is arbitrary, a useful approach to determining alpha for particular networks is through a hierarchical clustering algorithm. A hierarchical cluster analysis will present a succession of alternative, increasing, values of alpha. The first value of alpha will be zero. The actors who occupy structurally equivalent positions when alpha equals zero are equivalent under the definition of strong equivalence given in equation (2). Subsequent values of alpha are then obtained by combining individual positions of actors into subsets of positions which are more similar to each other than they are to other actor positions. This process continues by gradually allowing greater and greater social distances between actors clustered together as being structurally equivalent. Under the final value of alpha for a network, all actors will be clustered together into a single network position—even though they are separated by considerable social distances. Somewhere between the extreme of a definition of strong equivalence and the extreme of defining every position as equivalent, will be a reasonable partitioning of the actor positions into $M + 1$ sets of positions; M structurally nonequivalent network positions each jointly occupied by multiple, structurally equivalent, actors and one residual group of actor positions each occupied by no more than two actors. *Each of the* M *structurally nonequivalent network positions jointly occupied by multiple actors will appear in a hierarchical cluster analysis as a set of actor positions which are clustered together as equivalent when alpha is close to the definition of strong equivalence–i.e., demanding minimal difference between* ID* *vectors clustered together as equivalent–and which are not clustered with other positions of actors until alpha is extremely weak–i.e., allowing almost any difference between* ID* *vectors.*

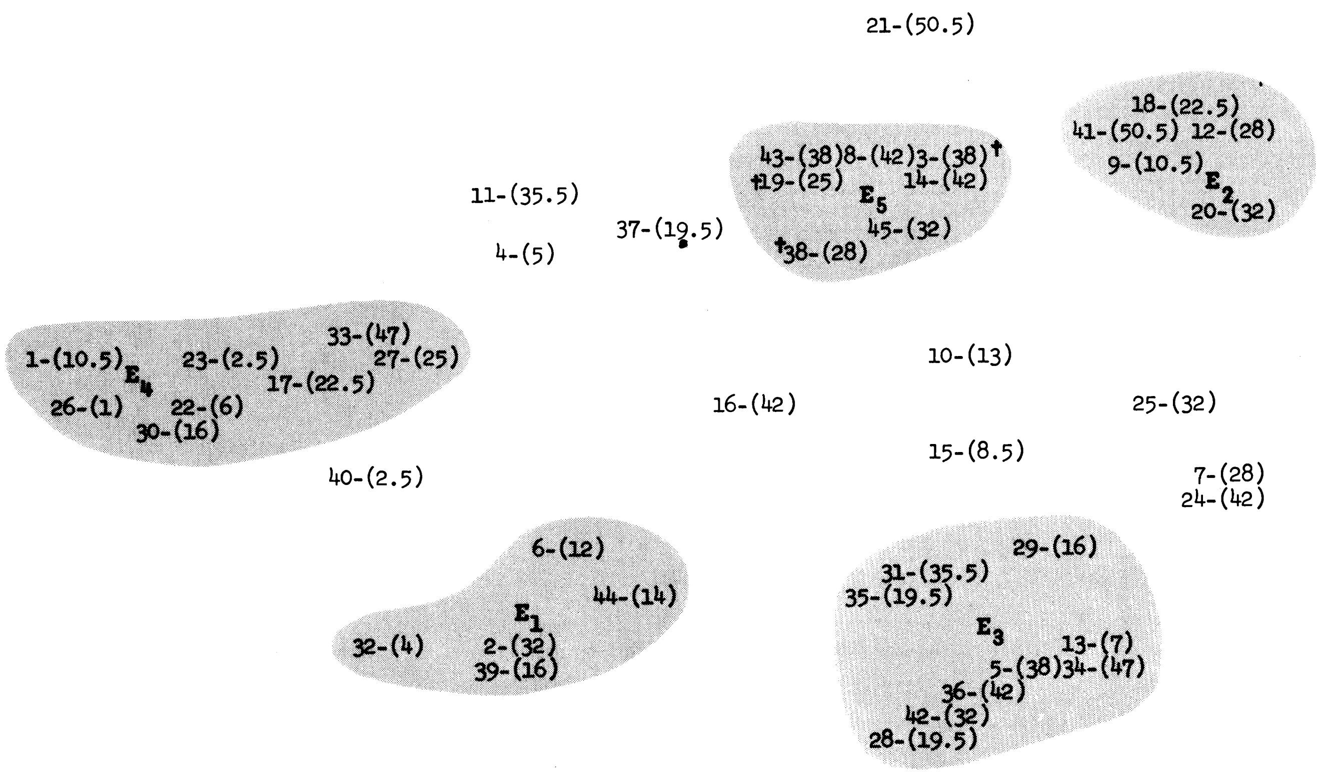
As an illustration of the ideas of strong and weak structural equivalence, Figures 1, 2 and 3 present analyses of the networks of economic exchange, social exchange and information-seeking among 45 community decision-makers (cf., Laumann and Pappi, a: Figures 3, 4 and 5 respectively). An (N by N) matrix of social distances as estimated from equation (1) using the operationalization of individual distance outlined in Appendix A was input into Johnson's connectedness method of hierarchical cluster analysis which is presented in part A of the figures and input into Roskam and Lingoes' smallest space analysis in two dimensions which is presented in part B of the figures. The location of network positions jointly occupied by multiple actors is based solely upon the hierarchical cluster analysis. The smallest space analysis is presented in order to give the reader a feel for the spatial distribution of actors in two dimensions and for comparison purposes with the original Laumann and Pappi discussion. Part B references actor positions by their sequential number as is done in part A of the figures as well as by the influence rank of the actor as presented in the Laumann and Pappi figures. Appendix B details the expected differences which will occur in a nonmetric analysis of social distances as opposed to individual distances using Figures 1, 2 and 3 versus Figures 3, 4 and 5 in Laumann and Pappi (a) as an example.

Figure 1. LOCATING CLUSTERS OF POSITIONS IN THE ECONOMIC EXCHANGE NETWORK.*

A. Hierarchical cluster analysis of the positions of the actors in the exchange network where "X" between positions indicates that the two positions are placed in the same cluster:



B. Smallest space analysis in two dimensions of the positions of the actors in the exchange network where the coefficient of alienation is .159 and a two factor principal components solution for the similarities between the positions of actors explains 60% of the variability in positions:

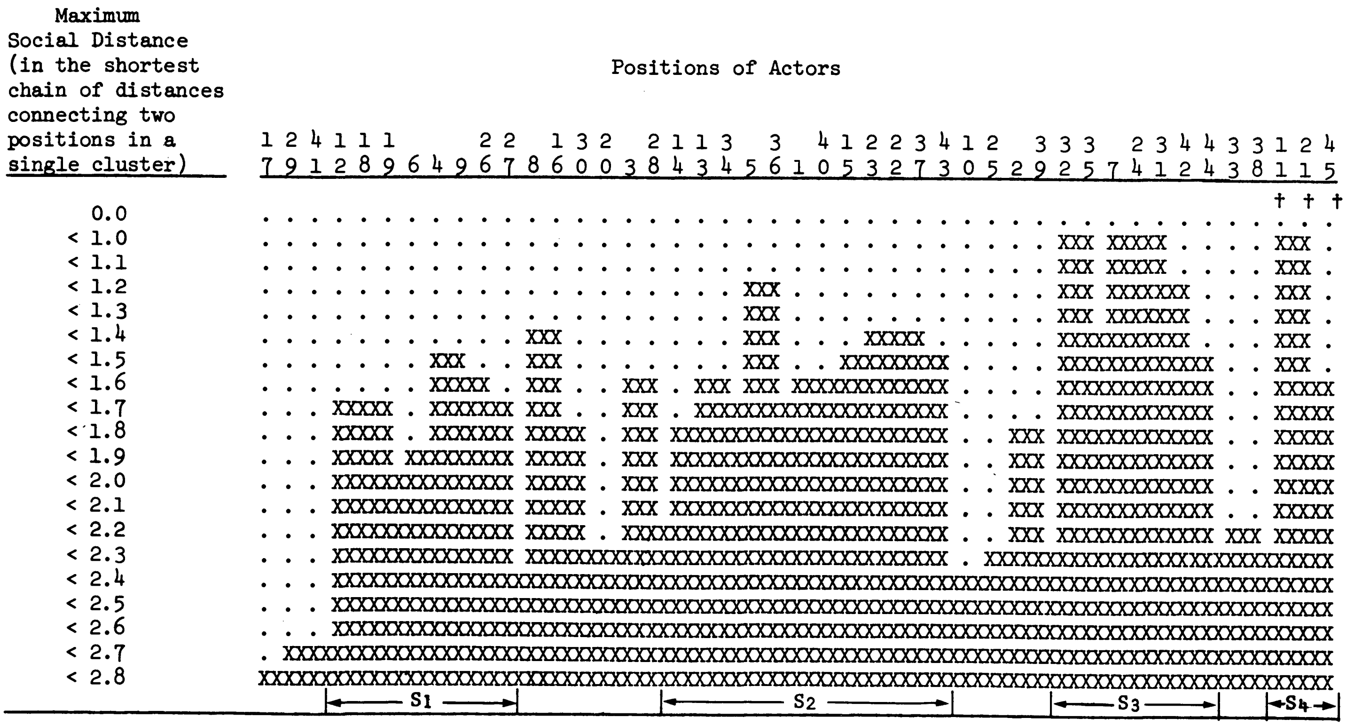


†Marks positions of actors selected as indicants of the structurally nonequivalent multiple actor positions

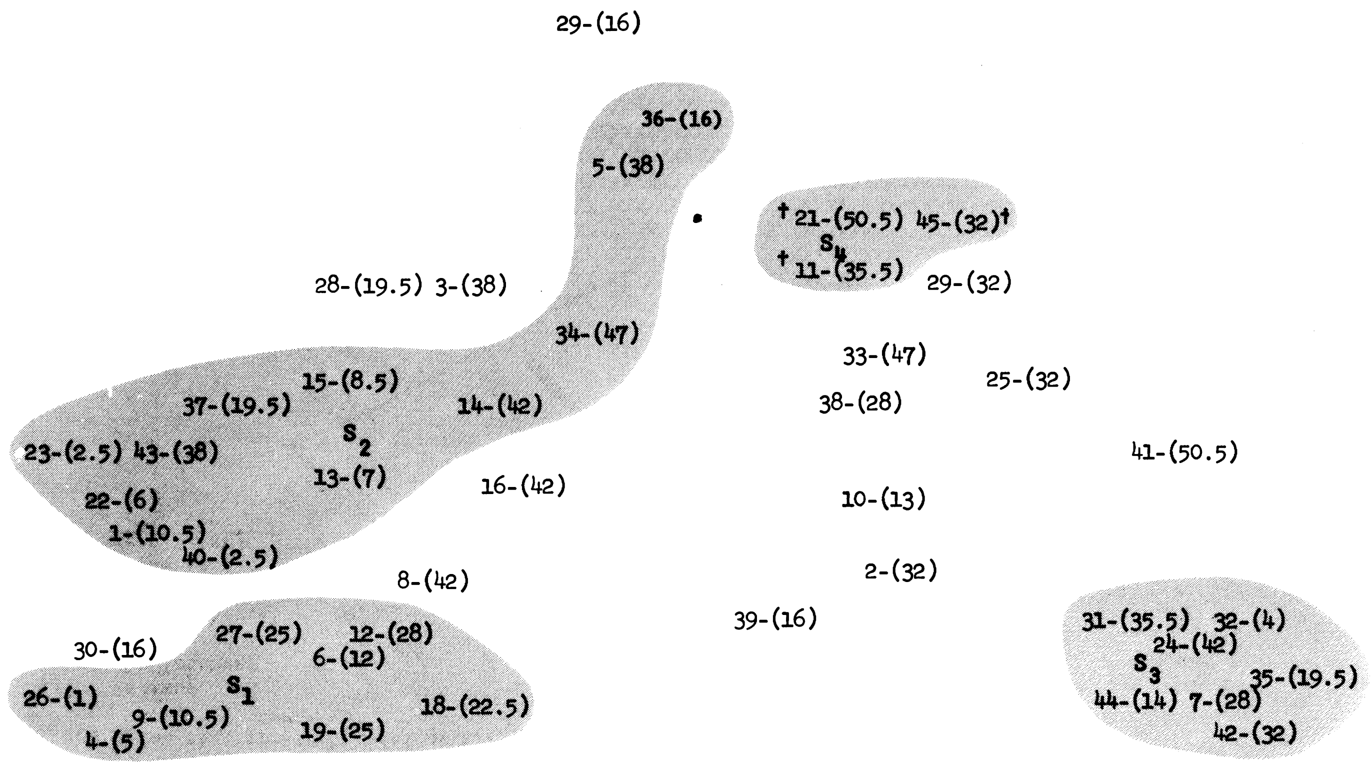
***Analyses based on binary sociometric responses to the question; "Could you now indicate the three persons out of our list with whom you have the closest business or professional contact?"**

Figure 2. LOCATING CLUSTERS OF POSITIONS IN THE SOCIAL EXCHANGE NETWORK.*

A. Hierarchical cluster analysis of the positions of the actors in the exchange network where "X" between positions indicates that the two positions are placed in the same cluster:



B. Smallest space analysis in two dimensions of the positions of the actors in the network where the coefficient of alienation is .159 and a two factor principal components solution for the similarities between the positions of actors explains 60% of the variability in positions:

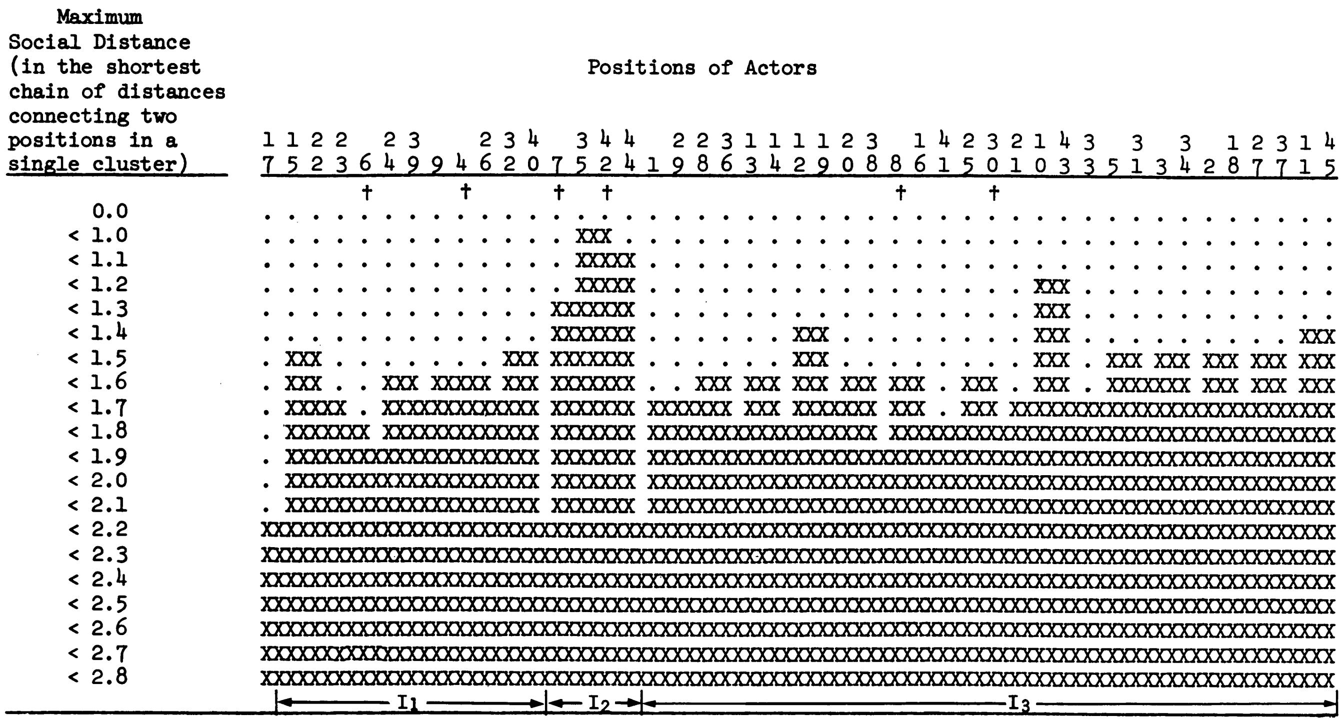


†Marks positions of actors selected as indicants of the structurally nonequivalent multiple actor positions

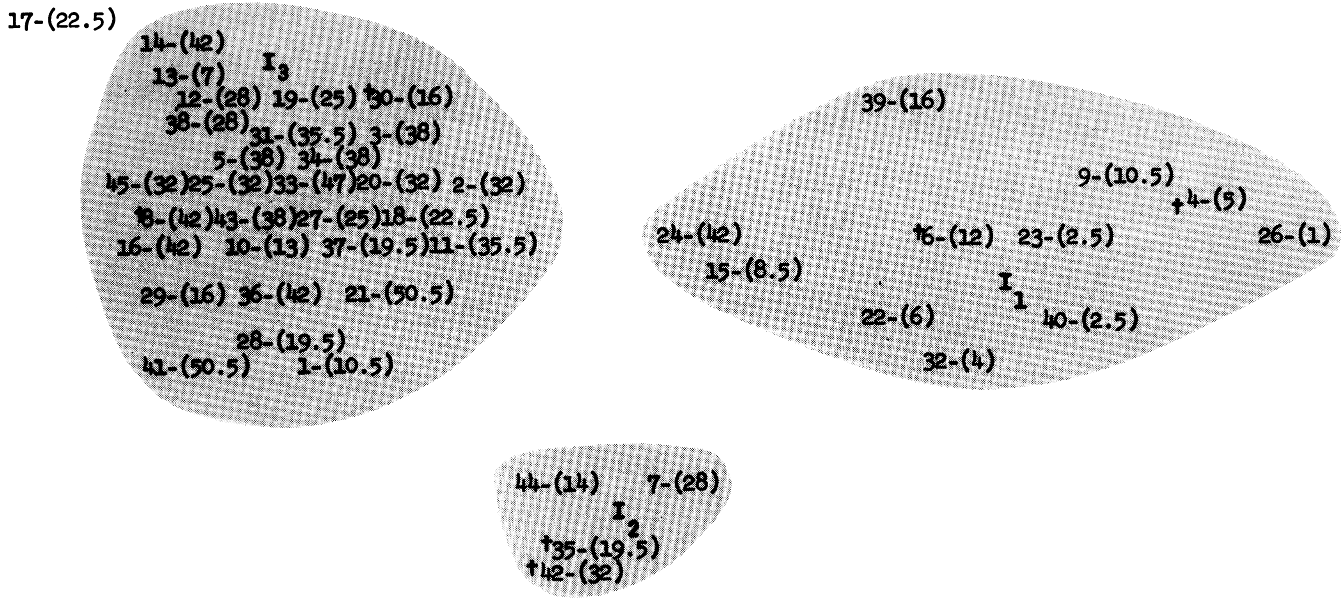
*Analyses based on binary sociometric responses to the question; "Would you please indicate the three persons from the list with whom you most frequently meet socially?"

Figure 3. LOCATING CLUSTERS OF POSITIONS IN THE INFORMATION SEEKING NETWORK.*

A. Hierarchical cluster analysis of the positions of the actors in the network where "X" between positions indicates that the two positions are placed in the same cluster:



B. Smallest space analysis in two dimensions of the positions of the actors in the network where the coefficient of alienation is .142 and a two factor principal components solution for the similarities between the positions of actors explains 79% of the variability in positions:



† Marks positions of actors selected as indicants of the structurally nonequivalent multiple actor positions

*Analyses based on binary sociometric responses to the question; "Could you please indicate the three persons with whom you most frequently discuss community affairs?"

Each row in the part A of Figures 1, 2 and 3 corresponds to a different value of alpha given on the left-hand side of the figure. Positions of actors in a network which are clustered together as being equivalent at a given level of alpha are so indicated by Xs connecting them (e.g., the positions of actors 35 and 42 in the information seeking network are to be considered equivalent when alpha is greater than 1.0).

The first value of alpha is zero. No positions in any of the three networks are equivalent under the definition of strong equivalence. In each of the figures, the last and highest value of alpha allows all network positions to be treated as equivalent. Following the above discussion, the M structurally nonequivalent positions jointly occupied by multiple actors will appear in part A of the figures as mounds of Xs separated by sharp dips or valleys between mounds. Such a condition will evidence a clustering together of actors when alpha is close to the definition of strong equivalence and the lack of additional actors positions entering the cluster until alpha is so large as to allow considerable social distances among actors clustered together as weakly equivalent. For example in the network of economic exchange relations (Figure 1) actors 2 and 39 are clustered together when alpha is 1.0 and other actors are clustered with them until alpha equals 1.5. After that, no actor is specified as structurally equivalent to the set composed of actors 32, 6, 2, 39 and 44 until alpha equals 2.5—almost the largest value given to alpha for the network. The mound of Xs in the columns associated with these actors indicates that these five actors jointly occupy a position in the network which is structurally unique from the positions of other actors and so has been identified as position E_1 in the analysis. Actors jointly occupying the M structurally unique positions in each network are indicated beneath their columns in the hierarchical cluster analysis and are circled in the smallest space analysis (actors in residual category are unlabelled). The five structurally unique positions in the network of economic exchange relations (i.e., $M = 5$) are referenced as E_1, E_2, E_3, E_4 and E_5 (e.g., the actors jointly occupying position E_2 are 9, 12, 18, 20 and 41). The four structurally unique positions located in the network of social exchange relations are referenced as S_1, S_2, S_3 and S_4 (i.e., $M = 4$). The three structurally unique positions located in the network of information seeking relations are referenced as I_1, I_2 and I_3 (i.e., $M = 3$).

CHARACTERIZING THE STRUCTURE IN AN OBSERVED NETWORK OF RELATIONS

The observed position of an actor in a network exists as the pattern of individual distances to and from actors in a system, i.e., as a particular form of interaction, given a single reason for interaction, i.e., given a particular content of interaction (see Simmel, 40–57, for an elaboration of the content-form distinction). Empirically, structure in a network exists as N observed positions in the network; ID_1, ID_2, . . . ID_n. At a higher level of abstraction, idiographic positions (positions whose

Table 1. STRUCTURE IN THE OBSERVED NETWORKS PORTRAYED AS THE PATTERN OF RELATIONS AMONG M+1 STRUCTURALLY NONEQUIVALENT SETS OF ACTORS*

Economic Exchange Network (Figure 1)

	E_1	E_2	E_3	E_4	E_5	Residual	Total	
E_1	87	0	0	13	0	0	100%	(15 citations)
E_2	0	0	0	0	0	0	100%	(0 citations)
E_3	12	4	56	8	0	20	100%	(25 citations)
E_4	0	0	0	90	0	10	100%	(21 citations)
E_5	5	0	5	55	13	22	100%	(22 citations)
Residual	22	0	17	39	0	22	100%	(18 citations)
Total	21	1	18	42	3	16	100%	(101 citations)

Social Exchange Network (Figure 2)

	S_1	S_2	S_3	S_4	Residual	Total	
S_1	81	5	0	0	14	100%	(21 citations)
S_2	11	78	0	0	11	100%	(35 citations)
S_3	11	0	72	0	17	100%	(18 citations)
S_4	11	22	0	33	34	100%	(9 citations)
Residual	37	23	3	0	37	100%	(35 citations)
Total	37	32	12	3	16	100%	(118 citations)

Information Seeking Network (Figure 3)

	I_1	I_2	I_3	Residual	Total	
I_1	97	3	0	0	100%	(31 citations)
I_2	33	67	0	0	100%	(9 citations)
I_3	72	3	24	1	100%	(76 citations)
Residual	0	0	0	0	100%	(0 citations)
Total	76	8	15	1	100%	(116 citations)

*Cell entries are proportion of citations made by actors in row to actors in column position. Total number of citations made by actors in row is at right.

particular mixture of form and content of interaction are of special significance for the system of actors being considered) and nomothetic positions (positions defined solely as forms of interaction) present alternative perspectives on structure in a network.

As a first cut, those positions which are jointly occupied by multiple, structurally equivalent actors are of special significance within a network. The definition of weak structural equivalence provides a means of aggregating the N observed positions of actors in a system into $M + 1$ structurally nonequivalent subsets of actors. Idiographically, basic characteristics of empirical structure in a network are preserved as M specific patterns of relations to and from each of the N actors (idiographic positions) and a conglomeration of various patterns of relations associated with the actors comprising the residual category of actors. The decision to treat positions in the residual category as idiographic will be based on conceptual grounds since it has no clear resolution on empirical grounds. The structure in an observed network of relations can then be characterized as the pattern of relations among the $M + 1$ sets of actors. Such representations of the structure in the networks analyzed in Figures 1, 2, and 3 are presented in Table 1. Table 1 shows the distribution of sociometric choices made by actors in each subset over the $M + 1$ structurally nonequivalent subsets of actors. For example, the network of economic exchange relations is presented in Table 1 as a pattern of relations among six sets of actors—five sets of actors where each set jointly occupies a unique idiographic position in the network and one residual set of actors. The actors in position E_1 give 87 percent of their sociometric choices to other actors in the same network position. The remaining 13 percent of their sociometric choices are given to actors occupying position E_4. This characterization of the structure in a network is similar to the idea of a blockmodel of a network (see White et al.) with the exception that actors are not forced into one or another structurally unique block. Each actor occupies his own network position. That position has finite social distances from each of the M structurally unique idiographic positions in the network.

Just as each of the M idiographic positions in a network is reflected in the empirical position of an actor as a function of the actor's social distances to actors jointly occupying each idiographic position, so the actor's empirical position reflects pure forms of interaction, i.e., nomothetic positions. Viewing the M structurally nonequivalent, jointly occupied, idiographic positions as basic elements of the structure in a network, one can, as an alternative to the characterization in Table 1, discuss the idiographic positions in terms of the pure forms of interaction to which each is most similar. Nomothetically, basic characteristics of empirical structure in a network can then be preserved as M types of pure forms of interaction.

Table 2 presents an exploratory typology of four forms of interaction. Each cell of the typology represents a different nomothetic position in a network. The positions are classified according to the prominence of actors within a network (zero versus non-zero proportions of sociometric choices made by actors in the network which are given to actors jointly occupying a position) and the tendency for actors to only initiate interaction with actors to whom they are structurally equivalent (half or over versus under half of the choices made by actors jointly occupying an idiographic position are given to other actors occupying the same position). The columns distinguish between actors in a position who receive few or no choices from the system versus those that receive a nonnegligible proportion of those choices. The rows in Table 2 distinguish between actors in a position who give most of their citations to other actors in the position versus those who give most of their citations to actors occuping different positions from their own. The four nomothetic positions in Table 2 are not intended to be exhaustive of the range of possible empirical positions, however, they do represent basic types of network positions as pure forms of interaction.

The isolate position exists as a set of actors who give most of their citations

Table 2. FOUR ELEMENTARY TYPES OF NETWORK POSITIONS AND THE PATTERNS OF RELATIONS WHICH DEFINE THEM

		Proportion of network choices given to the position*	
Ratio of position choices given to the position over total number of choices made by the position†		~.0	>.0
	≥.5	ISOLATES A	PRIMARY POSITION B
	<.5	C SYCOPHANT	D BROKER

ILLUSTRATIVE PATTERNS OF RELATIONS‡

		A	B	C	D	RESIDUAL
ISOLATE	A	0§	0	0	0	0
PRIMARY	B	0	high	0	0	0
SYCOPHANT	C	0	high	low	0	0§
BROKER	D	0	high	0	low	0§
	RESIDUAL	0	high	low	high	0§
	TOTAL	0	high	low	high	0§

*Proportion of all choices made by actors in the network which are given to the actors jointly occupying the network position being classified.

†Number of choices made by actors jointly occupying the network position being classified to actors within the position over the total number of choices made by actors occupying the position.

‡Cell entries refer to the proportion of sociometric choices from actors occupying the row position which are given to actors occupying the column position as is presented in Table 1.

§No predictions are made for these cells. Although they aid in the description of network structure, they have not been used in this exploratory classification of types of positions in a network.

to actors within the set and receive no citations from actors in the system who are not in the set, i.e., who occupy nonequivalent positions in the network. Such a set of actors could be a subsystem within the overall system of N actors or could be a group of actors each of whom is an isolate within the system (this latter possibility assumes that at least one isolate made a sociometric choice to himself or to an actor occupying the isolate position). Position E_2 in Figure 1 and Table 1 is an example of an isolate position.

A primary position in a network exists as a set of actors who give most of their sociometric citations to actors with whom they are structurally equivalent and who receive a nonnegligible proportion of the total number of sociometric citations made by the N actors in the network. Using sociometric citations as a measure of the interest of the citor toward the citee, primary positions are composed of actors who have greater interest in actors within their position than in actors occupying nonequivalent positions although they are the object of nonnegligible interest from the overall system of N actors as represented in the network being analyzed. Several examples of primary positions are illustrated in Table 1; E_1, E_3, E_4, S_1, S_2, S_3, I_1 and to some extent I_2.

The remaining two types of nomothetic positions distinguished in Table 2 (cells C and D) are usually grouped together with primary positions when a network is analyzed in terms of individual distances that are forced to be symmetric. These two types of positions are characteristically occupied by actors who cite prestigious actors in primary positions but do not have their choices reciprocated. These positions, labelled "sycophant" and "broker" in Table 2, are occupied by the usual hangers-on associated with prestigious or wealthy actors.[13] The term "sycophant" is perhaps too harsh here, however, it emphasizes the idea of sociometric choices being given to actors outside the position and lack of choices being given to actors in the position by actors outside the position. A broker position in a network will also be jointly occupied by actors who give most of their citations to nonequivalent actors, however, this position differs from the sycophant position in that actors in the broker position receive a nonnegligible proportion of citations from actors in the overall network. While actors in a broker position are the object of a nonnegligible proportion of the interest of actors in the system being considered, actors in a sycophant position are not.[14] Although there are no examples of broker positions in the networks presented in Table 1, there are two examples of sycophant positions; E_5 and I_3. The actors in position E_5 give 55 percent of their sociometric citations within the network of economic exchange relations to actors occupying position E_4. Only 13 percent of their citations are given to other actors in position E_5 and no one from position E_4 cites an actor in position E_5. Within the overall system, only 3 percent of the available citations are given to actors in position E_5. Actors in position I_3 give 72 percent of their citations to actors in the primary position I_1 while no one in positions I_1 or I_2 gives information-seeking sociometric citations to actors occupying I_3.

In summary, structure in a network of observed relations can be characterized either as a pattern of relations among M structurally nonequivalent, idiographic positions and a residual category of nonequivalent empirical positions of actors as is done in Table 1, or it can be characterized in terms of the reflection of forms of interaction, nomothetic positions, in the idiographic positions.[15] Under the latter characterization, the networks in Table 1 would be described as follows: the network of economic exchange relations consists of three primary positions, one isolate position, one sycophant position and a residual category of actors, the network of social exchange relations consists of three primary positions, a sycophant position and a residual category of actors, and the network of information-seeking relations consists of two primary positions, one sycophant position and a single isolate. The characterization in terms of idiographic positions as given in Table 1 is the more accurate, however, the characterization in terms of nomothetic positions emphasizes the forms of interaction in a network. The choice between the alternative characterizations can only be made based on the purposes of a given research project.

POSITIONS IN NETWORKS AS UNOBSERVED VARIABLES IN STRUCTURAL EQUATIONS

More important than their utility as a means of characterizing the structure in an observed network of relations, the differentiation of distinct types of network positions sets the stage for making statements about the expected antecedents and consequences of actors occupying various network positions. To the extent that a particular network plays a significant part in some social phenomenon, occupancy of different types of positions within the network should be associated with different antecedents and consequences. Related to the questions why particular network structures should be observed given specific conditions in a system of actors is the problem of measurement error in relational data. (See Holland and Leinhardt, c, for a detailed discussion of potential consequences for a network analysis of error in relational data.) All three problems (the etiology of network structure, the consequences of network structure and the measurement error usually associated with data) can be rigorously investigated using the ideas presented in the previous sections.

Let D_{nn} be the (N by N) symmetric matrix of social distances among the N actors in a network. Column j of the matrix, D_j, is a (N by 1) vector of the social distances of each actor from the network position occupied by actor j. The set of N different column vectors in D_{nn} can be partitioned into $M + 1$ sets corresponding to the $M + 1$ structurally nonequivalent sets of actors located in the hierarchical cluster analysis. Each of the vectors of social distances to actors within one of the M sets of structurally equivalent actors can now serve as an indicator of the "true" vector of social distances separating each actor from the jointly occupied position. The idea here is that there is an unobserved, true vector of social distances separating each actor from a jointly occupied position in the network and that this unobserved vector is responsible for the observed variation in each of the indicator vectors of social distances. Just as socioeconomic status can be conceptualized as an unobserved variable with level of completed education and dollars of income serving as indicator variables, the sycophant position in the network of social exchange relations, S_4 in Table 1, can be conceptualized as an unobserved variable with the positions of actors 11, 21 and 45 serving as indicator variables. In algebraic form, this suggests three epistemic statements linking the unobserved variable to its indicators (where all variables are expressed as deviation scores):

$$D_{11} = (\delta_{11,S_4})S_4 + W_{11}, \tag{4a}$$

$$D_{21} = (\delta_{21,S_4})S_4 + W_{21}, \tag{4b}$$

$$D_{45} = (\delta_{45,S_4})S_4 + W_{45}, \tag{4c}$$

where W_{11}, W_{21} and W_{45} are vectors of error components in the observed social distances from actors 11, 21 and 45 respectively, $(\delta_{i,S4})$ is a covariance between the unobserved position S_4 and the observed position of actor i, and S_4 is the vector of "true" social distances separating each of the N actors from the unobserved sycophant position.

A (3 by 3) observed variance-covariance matrix, S, can be estimated from the three observed vectors: D_{11}, D_{21} and D_{45}. The matrix S can then be approximated by a predicted variance-covariance matrix, Σ, which is based on six unknown parameters associated with equation (4) (the variance of S_4 is assumed standardized in equation 5 in order to make the equation identified):

$$S \simeq \Sigma = \begin{bmatrix} \delta_{11} \\ \delta_{21} \\ \delta_{45} \end{bmatrix} [\phi^2{}_{S_4}](\delta_{11}\delta_{21}\delta_{45}) + \begin{bmatrix} \theta^2{}_{W_{11}} & 0 & 0 \\ 0 & \theta^2{}_{W_{21}} & 0 \\ 0 & 0 & \theta^2{}_{W_{45}} \end{bmatrix}, \qquad (5a)$$

or, expressed in matrix notation:

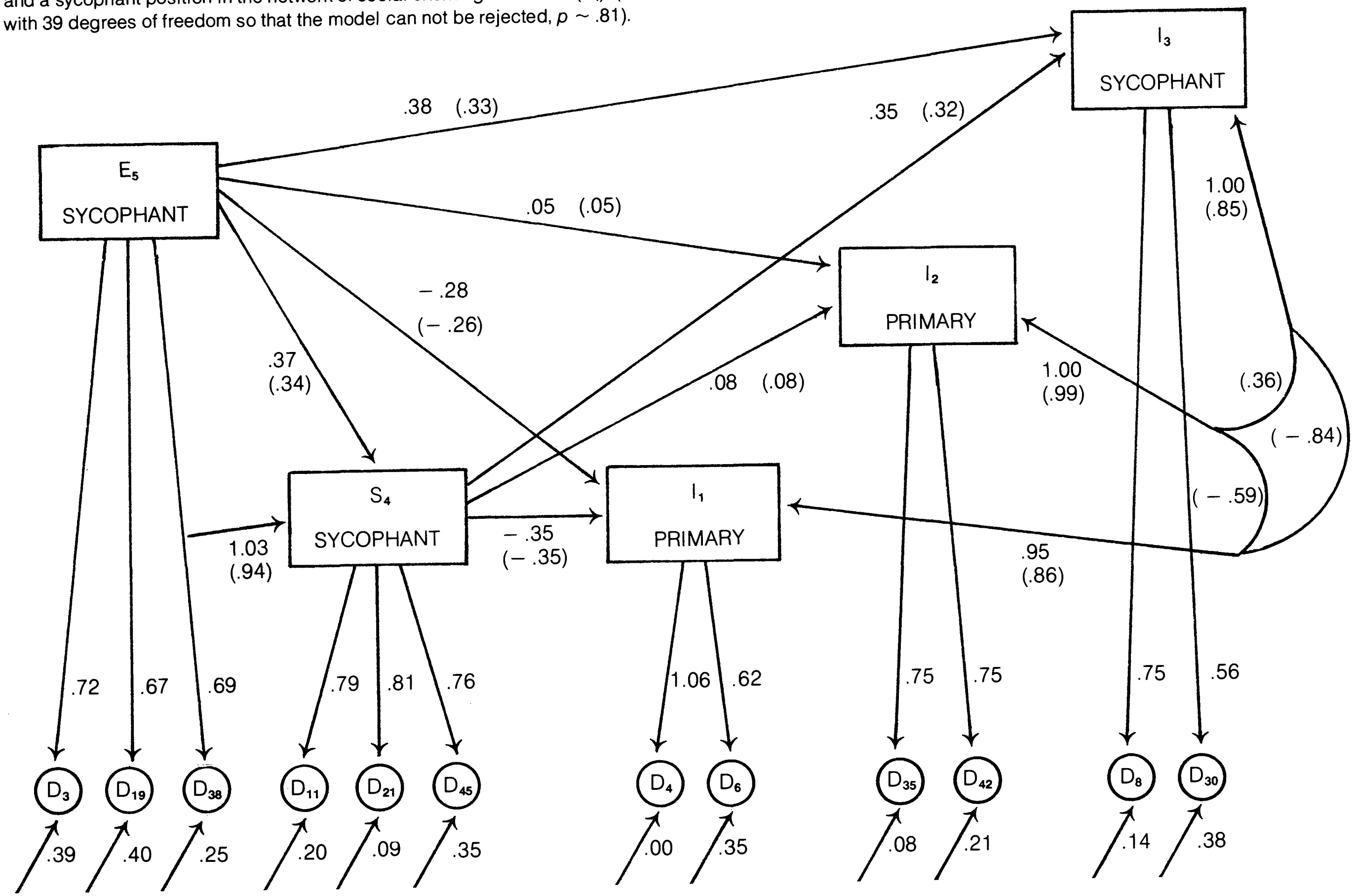
$$S = \Sigma = \Delta[\Phi]\Delta' + \Theta^2, \qquad (5b)$$

where the subscripts to the delta coefficients, δ, have been abbreviated to include only the indicator number since they all contain S_4, $\Phi^2{}_{S4}$ is the variance—here standardized—of the vector of "true" social distances separating actors in the network from position S_4, and $\Theta^2{}_{W_{11}}$, $\Theta^2{}_{W_{21}}$, $\Theta^2{}_{W_{45}}$ are variances of the error scores in the vectors W_{11}, W_{21}, W_{45}. Equation (5) is the basic factor analysis model for which a variety of estimation procedures are easily available (see e.g., Mulaik).[16]

The confirmatory factor analytic model has the same form as the unrestricted model in equation (5), except that specific unknown parameters in Δ, Θ or Φ can be forced to equal a priori values. (See e.g., Joreskog, a; Lawley and Maxwell; Mulaik.) Joreskog (b) extends the multiple factor, confirmatory factor analytic model such that causal inferences can be drawn among unobserved variables based on the observed covariances among the indicator variables. (See Joreskog and van Thillo for the computer program associated with the model and Burt, c; Werts et al.; Alwin and Tessler for applied discussion of the model.) Assuming a population multivariate normal distribution on the indicators, maximum-likelihood estimates are obtained for unknown parameters in the model and a chi-square statistic is routinely available which assesses the significance of the difference between S and Σ and is distributed with degrees of freedom equal to the number of overidentifying restrictions on the model being estimated.[17]

As an illustration of the investigation of the antecedents and consequences of aspects of network structure via structural equation models, consider the diagram in Figure 4. In Figure 4, squares enclose unobserved variables representing idiographic positions (labeled with the nomothetic position to which each is most similar) and circles enclose the observed positions of actors. E_5 is the sycophant position in the observed network of economic exchange relations. S_4 is the syco-

Figure 4. AN ILLUSTRATIVE STRUCTURAL EQUATION MODEL Specifying social distance from positions in the network of information seeking relations (I_1, I_2, I_3,) as a function of social distance from a sycophant position in the network of economic exchange relations (E_5) and a sycophant position in the network of social exchange relations (S_4), (standardized estimates are given in parentheses and $X^2 = 31.2$ with 39 degrees of freedom so that the model can not be rejected, $p \sim .81$).



phant position in the observed network of social exchange relations. I_1, I_2 and I_3 are the M structurally unique, jointly occupied network positions in the network of information seeking relations. The observed positions of actors 3, 19 and 38 in the network of economic exchange relations are specified as indicators of E_5 (see Figure 1). The observed positions of actors 11, 21 and 45 in the network of social exchange relations are specified as indicators of S_4 (see Figure 2). Similarly, observed positions in the network of information seeking relations are specified as indicators of their respective network positions (see Figure 3).

Figure 4 illustrates the idea of specifying jointly occupied network positions as unobserved variables in structural equation models. Its generalizability is negligible since the structural equations determining S_4, I_1, I_2 and I_3 have no theoretical rationale,[18] and the model (a restricted factor analysis with the factor dispersion matrix unconstrained) has to be further restricted in order to insure unique unstandardized parameter estimates of other than the error variances (the standardized estimates are unique). As a way of thinking, however, Figure 4 deals with an interesting phenomenon and one that is difficult to analyze using more traditional conceptualizations. It is hypothesized that being a sycophant in the network of economic exchange relations leads an actor to occupy a sycophant position in the network of social exchange relations. Occupying a sycophant position in either the network of social or the network of economic exchange relations then affects the type of position the actor occupies in the network of information seeking relations.

Table 3 presents the correlations and standard deviations from which the unknown parameters in Figure 4 have been estimated. Table 4 presents the standardized and unstandardized parameter estimates for Φ and Δ.[19] The chi-square approximation indicates the adequate fit of the hypothesized model to these data.

The unobserved network positions in Figure 4 measure the social distance of an actor from each position in the metric of social circles where a distance of one social circle from actor i is defined as the number of sociometric choices away from actor i actor j must be in order for the probability of actor i not initiating interaction with j to equal 1.0 (see Appendix A). Accordingly, the path coefficient, P_{se}, leading from unobserved position E to unobserved position S measures the expected number of social circles of change in an actor's social distance from S which will occur as a result of an increase of one social circle in the social distance of the actor from position E. For example, if an actor in the network of economic exchange relations manipulated his exchange relations such that his position in the network moved one social circle away from the sycophant position, E_5, then other things equal, proceeding as if the coefficients in Figure 4 are unique, he would simultaneously move .37 social circles away from the position jointly occupied by sycophants in the network of social exchange relations ($P_{s_4e_5} = .37$). In the information seeking network, he would move .28 social circles *toward* the primary position I_1 ($P_{i_1e_5} = -.28$) he would remain relatively the same social distance from position I_2 ($P_{i_2e_5} = .05$) and he would move .38 social circles away from the

Table 3. CORRELATIONS AND STANDARD DEVIATIONS FOR THE OBSERVED POSITIONS IN THE EXCHANGE NETWORKS AND INFORMATION SEEKING NETWORK WHICH ARE SPECIFIED AS INDICATOR VARIABLES IN FIGURE 4*

	D_3	D_{19}	D_{38}	D_{11}	D_{21}	D_{45}	D_4	D_6	D_{35}	D_{42}	D_8	D_{30}
D_3	1.00											
D_{19}	.76	1.00										
D_{38}	.83	.82	1.00									
D_{11}	.31	.27	.33	1.00								
D_{21}	.30	.28	.32	.97	1.00							
D_{45}	.35	.29	.39	.90	.92	1.00						
D_4	-.34	-.33	-.36	-.41	-.44	-.48	1.00					
D_6	-.30	-.30	-.30	-.31	-.33	-.36	.89	1.00				
D_{35}	.09	.05	.08	.09	.10	.14	-.56	-.49	1.00			
D_{42}	.06	.02	.06	.08	.08	.13	-.54	-.46	.96	1.00		
D_8	.47	.34	.39	.40	.43	.45	-.87	-.72	.35	.34	1.00	
D_{30}	.31	.24	.26	.32	.35	.37	-.77	-.62	.29	.28	.86	1.00
standard deviations	.83	.79	.74	.89	.90	.91	1.16	.77	.76	.79	.90	.76

*Positions D_3, D_{19} and D_{38} are from the economic exchange network, D_{11}, D_{21} and D_{45} are from the social exchange network, and D_4, D_6, D_{35}, D_{42}, D_8 and D_{30} are from the information seeking network.

sycophant position I_3 ($P_{i_3 e_5} = .38$).[20] In a similar manner, the path coefficients leading from the sycophant position in the network of social exchange relations to the three jointly occupied positions I_1, I_2 and I_3 can be interpreted in terms of social circles.

COMMENTS

Using the idea of a position in a network, I have sought to extend the analysis of network structure into areas of investigation which heretofore have been difficult to handle with more traditional perspectives. Several advantages derive from the perspective outlined in the previous discussions. First, multidimensional asymmetric relational measures are expressed in a symmetric scalar of social distance which incorporates relations toward and from actors so that easily available cluster and factor analytic algorithms can be used to analyze asymmetric relational data (Section 1). Second, the concept of structural equivalence is weakened in order to allow for the possibility of measurement error in the observed positions of structurally equivalent actors (Section 2). Third, network structure is characterized by the relations among $M + 1$ structurally nonequivalent sets of actors in a network—where only M sets of actors jointly occupy network positions—so that each actor is discussed in terms of his social distance from the M jointly occupied network positions rather than forcing him to occupy one of them, i.e., rather than partitioning

Table 4. VARIANCE-COVARIANCE MATRIX, Φ, AND FACTOR LOADING MATRIX, Δ, FOR THE MODEL IN FIGURE 4*

	E_5	S_4	I_1	I_2	I_3
Φ =	1.04	(.34)	(-.38)	(.08)	(.44)
	.39	1.22	(-.44)	(.10)	(.43)
	-.43	-.54	1.21	(-.56)	(-.88)
	.08	.11	-.63	1.03	(.36)
	.53	.57	-1.14	.43	1.40
Δ =	.72 (.88)	.0	.0	.0	.0
	.67 (.87)	.0	.0	.0	.0
	.69 (.94)	.0	.0	.0	.0
	.0	.79 (.97)	.0	.0	.0
	.0	.81 (1.0)	.0	.0	.0
	.0	.76 (.92)	.0	.0	.0
	.0	.0	1.06 (1.0)	.0	.0
	.0	.0	.62 (.89)	.0	.0
	.0	.0	.0	.75 (.99)	.0
	.0	.0	.0	.75 (.97)	.0
	.0	.0	.0	.0	.75 (.99)
	.0	.0	.0	.0	.56 (.87)

*Standardized estimates are given in parentheses.

the network into nonoverlapping cliques (Section 3). Fourth, network structure is characterized by the elementary types of jointly occupied positions of which it is composed so that the etiology and consequences of occupying different types of positions in networks on concomitant social phenomena can be investigated (Section 3). And finally, network positions are specified as unobserved variables in structural equation models so that the flexibility, mathematical simplicity and precision of structural equation models can be brought to bear on questions of the etiology and consequences of network structure, to analyze the applicability of the usual no errors-in-variables assumption in network analysis, and to assess the adequacy of alternative hypothesized network structures as descriptions of observed networks of relations (Section 4).

In order to simplify the exposition here, a single network of relations has been assumed within a system of actors. Multiple network systems of actors are addressed elsewhere as a simple expansion of the conceptualization given here (Burt, 1976f). A computer program, STRUCTURE, is available in FORTRAN IV for the IBM 370 that inputs either a sociometric choice matrix as analyzed here or multiple networks of relations defined on any other grounds and outputs social distances, a hierarchical cluster analysis of empirical positions of actors in a network or across multiple networks, and various sociometric indices specified here in footnote fifteen and in Lin. Card images are output so as to be used in analyses of antecedents and consequences of an actor's empirical position in a system of actors.

NOTES

1. Let G_{ik} be a vector of actor i's scores on K variables which characterize some domain of attributes over which actor i is to be compared to other actors. Let G^i_{jk} be actor i's perception of actor j's scores on the same K variables. Then a composite score of the difference between actor i and actor j—as perceived by i—can be given as the Euclidean distance between the vectors G_{ik} and G^i_{jk} (See Cronbach and Gleser's discussion of weighting profile differences).
2. Burt (b) and Burt and Lin discuss the use of content analysis of archival records as a means of generating relational data over time which is based on the idea that the relations between actors in a system are reflected in the joint occurrence of actors in descriptions of events occurring in the system.
3. This is the standard sociometric data generated by questions such as: "Who are your best friends?", "With whom do you most often discuss community affairs?", "From whom do you most often purchase goods?", etc.
4. Stemming from the work of Festinger, Luce and Luce and Perry, matrix multiplication is often used to determine the smallest number of sociometric choice links required by actor i to reach actor j. If Z is an (N by N) matrix of binary sociometric choice data (from footnote 3), then the matrix of shortest possible choice link distances among actors can be found through the sum $Z^* = Z + Z^2 + \ldots + Z^{N-1}$, where nonzero elements in the ith power of the original matrix are set equal to i, each successive power of the original matrix has deleted from it any elements which were nonzero in lower powers of the matrix and z_{ii}^* always remains equal to zero. The longest chain of choice links connecting two actors in the network equals q choice links when Z^{q+1} has no nonzero elements which were nonzero in lower powers of the matrix.
5. In regard to the shortest number of sociometric choice links connecting actor i with actor j (z^*_{ij} in footnote 4), Katz (b) suggests that as choice links become further removed from actor i, they should have less significance for him than do choice links close to him. Katz suggests inserting a scale factor in the computation of Z^* which decreases as Z is raised to increasing powers. Hubbell outlines a weighting scheme which weighs choices from prestigious actors more heavily than choices from relatively insignificant actors.
6. See for example, the measure of individual distance outlined in Appendix A as the probability of actor i not initiating interaction with actor j. This operationalization is the one which is used in the forthcoming numerical illustrations.
7. Even when Id_{ij} is analyzed by a nonmetric clustering algorithm for bending data to a specified number of dimensions (e.g., Lingoes; Roskam and Lingoes; Young) the analysis will overlook the structural aspect of social distance due to the reliance by these methods on the criterion of monotonicity which is only concerned with the distance between pairs of actors—even though the entire (N by N) matrix is being analyzed. The inability of methods of analysis based on the criterion of monotonicity to consider indirect linkages among actors is succinctly discussed by Coleman.
8. Equation (1) makes two major assumptions about the relations in a network. First, it is assumed that the individual distance of an actor from himself is zero, ($Id_{ii} = 0$). When this is not the case, then equation (1) can be given as: $d^*_{ij} = \sqrt{[(ID_i^* - ID_j^*)'(ID_i^* - ID_j^*)]}$, which is quite different from equation (1). In order for actors i and j to have zero social distance in a network where $\{Id_{ii}\} \neq 0$, they must have zero differences between their respective relations with each other actor in the network and they must relate to each other in the same manner that they relate to themselves. Equation (1) is a less general statement of the same idea. If it is assumed the $\{Id_{ii}\} = 0$, then d_{ij} will only equal zero if actors i and j have zero individual distances between them and identical relations to other actors in the network. I have subtracted ID^2_{ij} and ID^2_{ji} from the equation in (1) since these terms appear twice in d^*_{ij} when $\{Id_{ii}\} = 0$. A second assumption made in equation (1) is that all relations among actors should be weighted equally. This assumption can be deleted if the equation is complicated still further. Let W be a ($2N$ by $2N$) diagonal matrix where $\{w_{ii}\} = \{w_{i+N,i+N}\}$ = the weight of the importance of actor i relative to the other actors in the network (e.g., prestige scores, power scores, etc.). The generalized form of equation (1) given above can be further generalized to take into account the differential importance of the actors in the network as $d^*_{i^*j} = \sqrt{[(ID_i^* - ID_j^*)' (W) (ID_i^* - ID_j^*)]}$. In order to weight different types of relations more heavily than others (e.g., to give null relations more importance over nonzero relations) all $2N$ diagonal elements can vary according to the difference term they weight. For example if zero individual distances were to be weighted as twice as important as nonzero relations, the diagonal elements in W could be defined as $\{w_{kk}\} = 2$ if Id_{ik} or Id_{jk}, Id_{ki} or Id_{kj} equal zero, otherwise it equals 1.
9. Katz (a), in a discussion of the general utility of treating sociometric choice data via matrix methods, suggests the cosine of the angle between the choice vectors of two actors as a measure of their similarity. This measure is the correlation between the rows in ID_{nn} when individual distance is operationalized as

given in footnote 3. If the vector is extended to be defined in terms of relations from other actors as well as to other actors, then Katz's idea could be applied here as the cosine between ID_i^* and ID_j^* as a symmetric measure of the social distance between actors *i* and *j*. I have instead selected the Euclidean distance in equation (1) because it does not force the linearity assumption on the quantification of social distance.

10. In a similar sense the idea of individual distances being transitive or balanced is invariant over research applications (see e.g., Davis, a, b; Hallinan; Heider; Holland and Leinhardt, a, b). The social distance between actors *i* and *j* can be seen as a measure of the dissonance or imbalance in the overall relation between actors *i* and *j* within a specific network. d_{ij} will increase as a function of the difference in relations between actors *i* and *j* and *all* actors in the network become different. Unfortunately, much of the work on transitivity within a triad does not consider the three person group in terms of the actors linked to each of the persons in a triad (see Flament; and Hallinan and McFarland for exceptions to this statement). In relation to the discussions of transitivity, equation (1) considers all *N* persons in a network in order to expand the distance between persons who are in intransitive triads or tetrads of *N*-person groups.

11. The definition of strong equivalence is taken from the basic concept of equivalence in typology (see e.g., Kelley, 9–10) discussed in sociology by Holland and Leinhardt (b) and Lorrain and White. Lorrain and White (63) suggest: "Objects *a*, *b* of a category *C* are structurally equivalent if, for any morphism *M* and any object *x* of *C*, *aMx* if and only if *bMx*, and *xMa* if and only if *xMb*. In other words, *a* is structurally equivalent to *b* if *a* relates to every object *x* of *C* in exactly the same way as *b* does." A morphism is any direct or indirect relation between two objects or actors. A category is a set of objects and a set of morphisms. In equation (2) the category over which two actors can be structurally equivalent consists of the set of *N* actors as represented in networks of some type of relations and the morphism of individual distance ($aMx = Id_{ax}$, $bmx = Id_{bx}$, $xMa = Id_{xa}$, $xMb = Id_{xb}$). In a related perspective, Holland and Leinhardt (b, 110–1) specify structural equivalence of actors in an *M*-clique when the *M*-clique is situated within a larger network of actors. The utility of stating structural equivalence in terms of the scalar in equation (2) is twofold: (1) its greater simplicity, and (2) its facile generalization to a weak criterion of structural equivalence as given in equation (3).

12. Reference the difference between two actors' relations to other actors in the network as "structural" distance. This stands in contrast to individual distances between the two actors themselves. Admitting the possibility of weak structural equivalence brings in problems of interpreting the conditions of individual and structural distance under which actors will be deemed equivalent . The only time that the values of individual and structural distance responsible for an observed social distance can be determined from the observed social distance with certainty is when d_{ij} equals zero. In such a case, all component distances have to be zero. In equation (3), however, an individual only knows that if d_{ij} is less than the criterion distance then structural and individual distances are also less than the criterion. Isolates will have zero structural distance between them but individual distances of 1.0 separating them so that the social distances among isolates will equal 1.41. This means that the isolates in a network will be weakly structurally equivalent whenever the criterion alpha is greater than 1.41 (e.g., in the economic exchange network in Figure 3, the isolates are simultaneously clustered together as position E_2 as soon as the criterion increases from less than 1.4 to less than 1.5). In short, special attention needs to be paid to the degree to which a chosen nonmetric solution has collapsed otherwise meaningful distances among actors. This caution extends to the clustering algorithms mentioned by White et al. in order to locate "blocks" of actors in a network—particularly in the case of solutions which are a "lean" fit (See Breiger et al.).

13. The idea of sets of actors being defined by their common reference to actors or events external to the set—as are the actors jointly occupying sycophant or broker positions as discussed here—has been discussed under various labels such as; "quasi-groups" (Ginsberg, 40), "latent groups" (Olson, 50), "levelling coalitions" (Thoden van Velzen, 241–2) and "latent corporate actors" (Burt, b).

14. Boissevain (147–69) provides an interesting discussion of some of the expected behaviors and characteristics of actors occupying what is here described as a broker position in a network.

15. This line of reasoning can be continued to express the extent to which the empirical position of an actor reflects each of the four nomothetic positions in Table 2. Define two scalars describing aspects of actor *j*'s empirical position in a network: A_j = the prominence of actor *j* within the network = $(N - \sum^{N}_{i=1} \mathrm{Id}_{ij}) / N = 1.0$ when all actors have maximum probability of initiating interaction with actor *j*; B_j = the tendency for actor *j* to initiate interaction only with actors structurally equivalent to himself = $\left[\sum^{N}_{i=1} \mathrm{Id}_{ji} d_{ji}\right] / \left[\sum^{N}_{i=1} d_{ji}\right] = 1.0$ when actor *j* only has maximum probability of initiating interaction with those actors who have zero social distance from him, where Id_{ij} and d_{ij} are respectively individual

(Appendix A) and social (equation 1) distance from actor i to actor j. The extent to which actor j occupies a primary position within the network can then be given as; $(A_j)(B_j)$. The extent to which actor j occupies a sycophant position in the network can be given as; $(1 - A_j)(1 - B_j)$. The extent to which actor j occupies an isolate position is given as; $(1 - A_j)(B_j)$. Finally, the extent to which actor j occupies a broker position is given as; $(A_j)(1 - B_j)$. Each of the indices equals 1.0 at its maximum and zero at its minimum. Since they are expressed without reference to particular relations between particular actors, these indices can be used as crude indicators of structural forms reflected in the empirical positions of actors in separate networks and separate systems.

16. MacRae and Bonacich discuss cliques in networks in terms of a factor analytic model. The present situation differs from theirs in two respects. First, use of social distances, equation (1), allows asymmetric relational data to be expressed in the form of a symmetric scalar so that easily available cluster analytic and factor analytic algorithms can be used to analyze the structure in a network. Second, while both MacRae and Bonacich explore possible methods of applying the factor analytic model to locate cliques, the present approach is using a nonmetric, nonlinear cluster analytic model to locate structurally nonequivalent positions and then using a confirmatory factor analytic model to assess the adequacy of the hypothesized structure. This second difference provides a method of assessing the significance of the lack of fit of a hypothesized clique structure or nonequivalent positional structure to an observed network as well as enabling an investigator to specify antecedents and consequences of network structure in structural equation models (see Figure 4).

17. The chi-square statistic is given as a function of the discrepancy between S and Σ;

$$\chi^2 = (N - 1)\,[\ln|\Sigma| + \text{trace}(\Sigma^{-1}S) - \ln|S| - r]$$

where r equals the total number of observed variables in a hypothesized structural equation model.

Since the social distance function estimates d_{ij} by taking into account other observations, however, there will be fewer than N independent observations underlying the chi-square statistic and the estimation of S. This, in combination with the usual small N in network analysis, means that the chi-square statistic is a very rough approximation and should only be taken as an indication of substantive significance when its value has a high probability of occurring or an extremely low probability of occurring.

18. I have in mind here the specification of the determinants of change in patterns of relations among actors in a network. How do jointly occupied, structurally nonequivalent positions come to exist in a network? Explanations along this line have been idiographic due to the emphasis in past network analyses on description. However, this promises to be a most fruitful area for investigation as several recent works emphasize. Jointly occupied positions could arise as a function of causal relations among networks based on different types of relations (e.g., Laumann et al.) as a function of social backgrounds (e.g., Barton et al.; Gans; Laumann; Laumann and Pappi,b), as a function of changes in environmental conditions which affect the system of actors (e.g., Allen; Burt, d, b, e; Stanworth and Giddens) or as a function of actors purposively exploiting their positions in exchange networks in order to realize goals (e.g., Boorman; Burt, d, e; Granovetter, a, b).

19. The structural coefficients among the unobserved variables in Figure 4 are regression coefficients which can be estimated from the variance-covariance matrix Φ using ordinary least squares since the model specifies no paths equal to zero. These estimates will be equivalent to those in Figure 4 which are maximum likelihood estimates generated from a confirmatory factor analytic model (programmed as LISREL, Joreskog and van Thillo). Since the emphasis in the present discussion is on network structure rather than structural equation models, I have avoided the statistical and algebraic complications involved in the general estimation and specification of coefficients among unobserved network positions beyond the bivariate covariances in equation (5). The reader interested in more detailed treatment is referred to Joreskog (b) for a general discussion or to Burt (a, c), Werts et al., and Alwin and Tessler for applied discussion.

20. I have ignored the second-order effects of the compound path coefficients measuring change in position in the information-seeking network resulting from change in the social exchange network. Change in E_5 leads to change in S_4 which in turn leads to change in I_1, I_2 and I_3. For example, an increase of one social circle of distance in E_5 would lead to an increase of $(P_{i_3e_5} + P_{s_4e_5}P_{i_3s_4}) = .38 + .13 = .51$ social circles of distance away from position I_3 instead of the .38 reported in the text.

APPENDIX A: AN OPERATIONALIZATION OF INDIVIDUAL DISTANCE AS THE PROBABILITY OF ACTOR I NOT INITIATING INTERACTION WITH ACTOR J

In the interest of stating individual distance in a metric which is interpretable across different types of networks (e.g., networks of friendship relations, of economic exchange relations, etc.) it would be convenient to have a measure of the probability of actor i not initiating interaction with actor j as an operationalization of Id_{ij} to be substituted in equation (1) of the text. In other words, if Id_{ij} equals 1.0 in a network of economic exchange relations, it shows that actor i will not seek economic goods from actor j as the network is currently arranged. The operationalization given here is based on the minimum number of choice links required by actor i to reach actor j ($z_{ij}*$ in note 4). It has the advantage of a metric interpretation in terms of the circle of actors linked to actor i within the network and has therefore been used in the text illustrations.

The operationalization of Id_{ij} as the above described probabilistic measure requires three pieces of information: (1) conditions leading Id_{ij} to equal its minimum, (2) conditions leading Id_{ij} to equal its maximum, and (3) a function relating change in Id_{ij} to change in $z_{ij}*$. Actor i will have a circle of actors with whom he has a nonzero probability of initiating interaction (cf. Kadushin's discussion of the concept of an actor's social circle). Within this social circle, actor i has the highest probability of interacting with himself. $Id_{ii} = 0$. There is a close to zero probability of interaction being initiated by actor i toward actors separated from him by an infinite $z_{ij}*$ (e.g., isolates in the network or persons in separate systems of actors). Id_{ij} can be treated as having reached its maximum when $z_{ij}*$ is larger than the largest finite $z_{ij}*$ for actor i. Let ζ_{ij} be the largest finite $z_{ij}*$ linking actor i to any actor in the network. Since values of $z_{ij}*$ which are larger than ζ_{ij} imply a zero probability of actor i initiating interaction with actor j, ζ_{ij} provides a boundary for the social circle around actor i. The value of ζ_{ij} can be discussed as the range of actor i's social circle. Actor i would not be expected to initiate interaction with actors outside his social circle unless he needs unusual resources not readily available from actors within his circle (e.g., see Granovetter, a; Lee; Milgram; White, for discussion of the small world phenomenon and the use of weak links to realize unusual interests). Given the above anchorings for the maximum and minimum values of Id_{ij} relative to $z_{ij}*$ the rate of change in Id_{ij} in relation to change in $z_{ij}*$ can be drawn from research on human perception. Findings in psychophysics demonstrate the limited ability of persons to retain information past a boundary level of stimulation and inverse ability to discriminate differences in levels of stimulation as the overall level of stimulation approaches the boundary level (see e.g., Haber; Simon). The level of stimulation here is the proportion of actors in his social circle with whom actor i will initiate interaction. It is to be expected that actor i would have little difference in Id_{ij} for the few actors who are close to him relative to the other actors in the network, that the change in Id_{ij} would increase with change in $z_{ij}*$ as a function of the number of actors at a distance $z_{ij}*$, and that actor i would have little difference in Id_{ij} for the actors who are near the boundary of his social circle and beyond. Let the N values of $z_{ij}*$ be rank ordered from lowest to highest for actor i: Change in Id_{ij} would then be a function of the cumulative number of actors in i's social circle who are less than or equal to $z_{ij}*$ choice links away from i:

$$Id_{ij} = \begin{cases} 0, \text{ if } i = j, \\ \dfrac{\text{cumulative frequency of } z_{ij}*}{[(\text{number of actors in social circle of actor } i) + 1]}, \text{ if } z*_{ij} \leq \zeta_{ij}, \\ 1, \text{ if } z_{ij}* > \zeta_{ij}. \end{cases}$$

Id_{ij} will be low (i.e., reflect a high probability of i initiating interaction with j) when $z*_{ij}$ is low relative to the other $z_{ik}*$ linking actor i to actors in the network. The important characteristic of the individual distance between actor i and j—in terms of the probability of i initiating interaction with j—is not the absolute number of choice links required by i to reach

j, but rather the location of actor j in actor i's social circle. The decision to interact with actor j can be seen as a marginal consumption decision. The commodity being consumed here is the initiation of interaction with a proportion of the actors in actor i's social circle. From an initial point of $Id_{ij} = .5$, increases in z_{ij}^* result in smaller and smaller increases in Id_{ij} until Id_{ij} reaches its maximum of 1.0. From the same point, decreases in z_{ij}^* result in smaller and smaller decreases in Id_{ij} until Id_{ij} reaches its minimum at 0. What this says is that as z_{ij}^* approaches its limits at either the boundary or center of actor i's social circle, the marginal propensity of actor i to initiate interaction with actor j declines.

APPENDIX B: CLUSTERS OF ACTORS IN NONMETRIC CLUSTERING ALGORITHMS BASED ON INDIVIDUAL VERSUS SOCIAL DISTANCES

It is common for individual distances among actors to be forced to be symmetric and then to be subjected to a nonmetric clustering algorithm—often a smallest space analysis (see review by Bailey). Given this frequent practice by sociologists, an important question in reference to the location of the M structurally nonequivalent positions in Section 3 concerns the different inferences which would result from a nonmetric cluster analysis of symmetrized individual distances instead of the social distances used to compare actors as network positions.

A comparison of the smallest space analyses in Figures 1, 2 and 3 with Figures 3, 4 and 5 in Laumann and Pappi (a) will illustrate the potentially different conclusions which can be gleaned from analyses based on individual versus social distances between actors. As a general rule of interpretation, an analysis based on individual distances (as exemplified by Laumann and Pappi's careful analysis) will conclude that two actors occupy similar positions in a network if they interact frequently or if they have homophily in terms of some profile of characteristics—i.e., if they have small individual distance between them. An analysis based on social distances (as illustrated here in Figures 1, 2 and 3) will conclude that two actors occupy similar positions if they have the above characteristic of small individual distance between themselves as well as having similar individual distances to and from the other actors in the network—i.e., if they have small social distance. Analyses of individual distances emphasize linkages between pairs of actors. Analyses of social distances emphasize similarities between patterns of relationships with the N actors in a system.

There are two general characteristics of a network analysis in terms of which an analysis of individual distances can be compared with one based on social distances: (1) the expressed structurally nonequivalent positions in a network, and (2) the distribution of actors in reference to those nonequivalent positions. Concerning the first characteristic, notice that Figures 1, 2 and 3, through the use of social distances, emphasize the similarity between actors in structurally equivalent positions and emphasize the differences between actors in nonequivalent positions (under the weak criterion). The structurally nonequivalent positions in a network are thereby made more obvious and easier to detect in an analysis. This emphasis on structure is most vividly demonstrated in Figure 3 where the previous scatter of actors through the smallest space analysis of individual distances (Figure 5 in Laumann and Pappi) is here reduced to three nonequivalent network positions. These structurally nonequivalent positions provide a set of reference points from which each actor in a network can be located and discussed in contrast to the idea of comparing actors in terms of their relative distance from the "center" of a smallest space. Concerning the second characteristic of a network analysis given above, actors are distributed in space with reference to nonequivalent positions according to the principle of interpreting an analysis based on individual distances versus one based on social distances which was emphasized above. For example, the isolates in the economic exchange network are occupying a joint position in the network instead of being scattered around the periphery of the network as they would be in an analysis of individual distances. Similarly, the six most influential actors in the community jointly occupy a position in the information-seeking network (position I_1 in Figure 3) because they all have the characteristic of receiving many citations from other actors in the network which they do not reciprocate. Their position in the network is placed on the periphery of the social space as represented in the smallest space analysis since they occupy a position which is very different from the positions of most actors in the network. In an analysis of individual distances these actors would be near the center of the space since so many actors claim to interact with them (this characteristic is elaborated by Laumann and Pappi, a, as a measure of the integrative centrality of the actor). In an analysis of social distances, on the other hand, an actor is close to the center of the social space when his position is equally dissimilar to the positions of the other actors in the network. The more that the actors jointly occupying a network position receive a disproportionate number of sociometric choices—more or less—then the more peripheral the position will be in the social space as represented in a smallest space analysis.

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